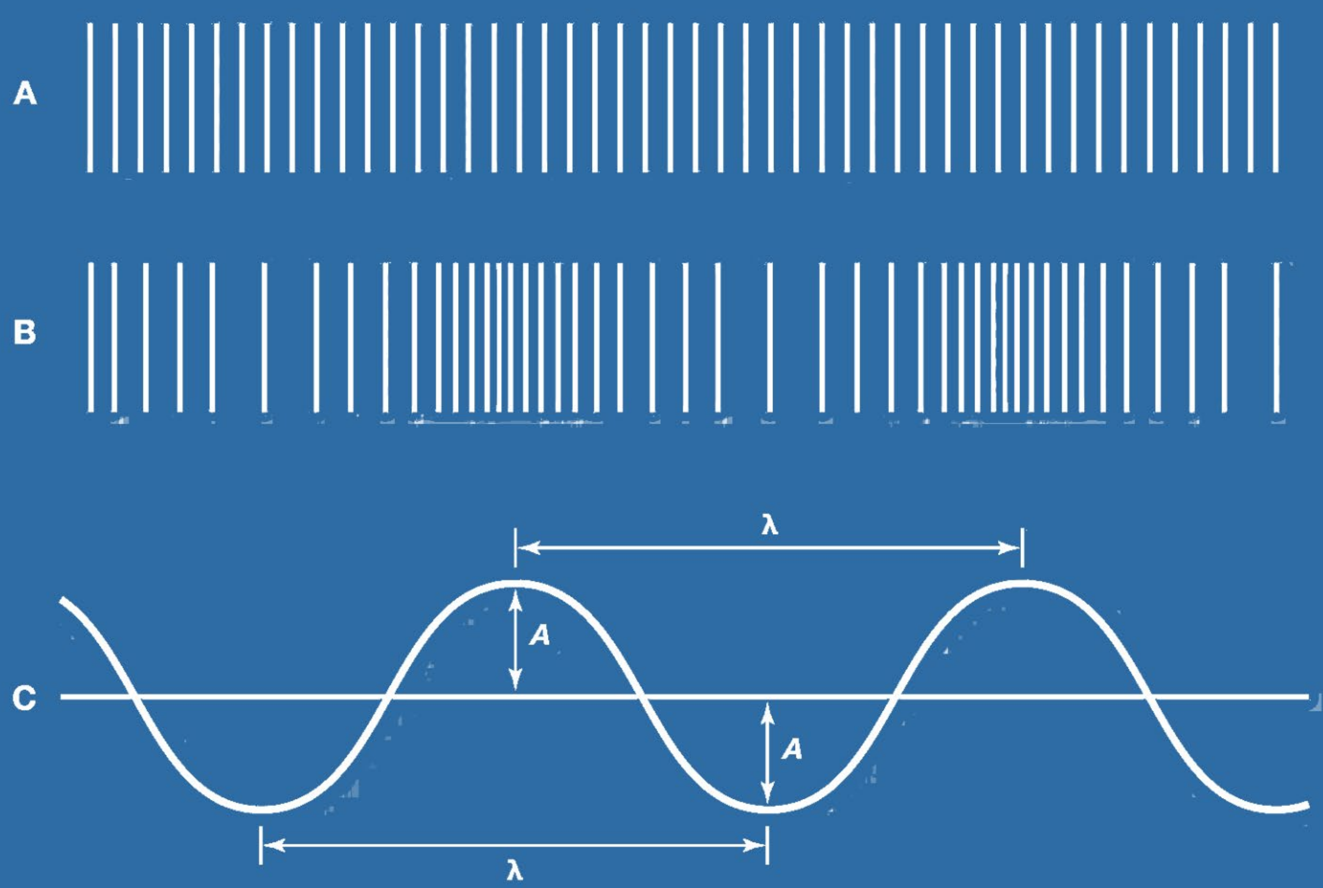
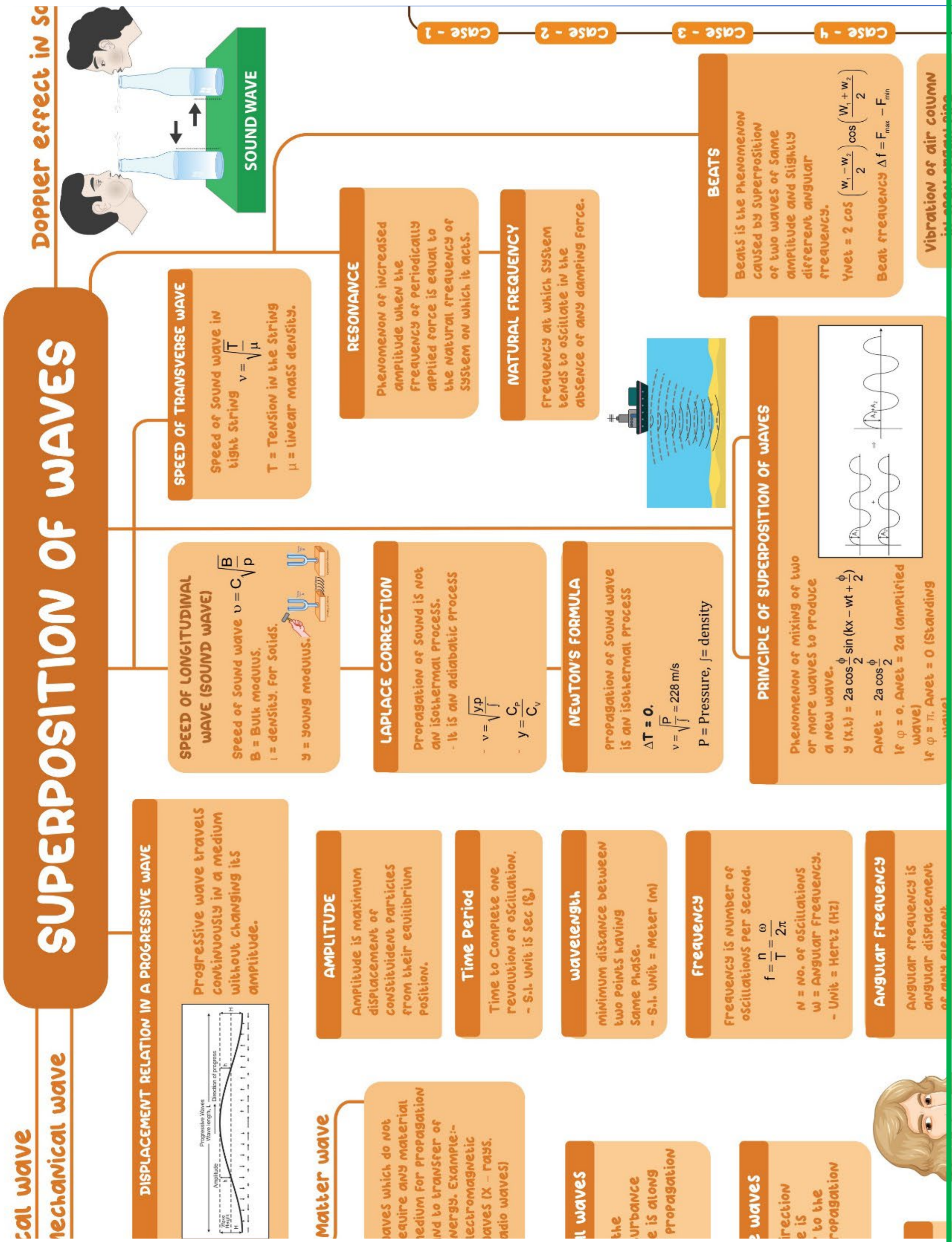
15.Waves



Ramanujan School of Mathematics and Physics

Theory + NCERT MCQs + Topic Wise Practice MCQs + NEET PYQs



Waves

Progressive (Mechanical) wave

When a source produces a disturbance in a medium, the particles in the adjacent layers pick up the disturbance and start vibrating. These in turn transfer the disturbance to the next layers and so on. This leads to propagation of disturbance away from the source. Such ***a continuous propagating disturbance is called a progressive wave***.

Classification of waves

1. Mechanical waves and electromagnetic waves

Based on the nature of disturbance which propagates, waves are classified as

- **Mechanical waves: waves which are elastic disturbances and *require a material medium* for their propagation, are called mechanical waves**.

In the case of mechanical waves, particles in the medium oscillate without drifting along the direction of wave propagation.

Example:**Sound waves, seismic waves and shock waves.**

- **Electromagnetic waves: waves which are the disturbances in electric and magnetic fields and *donotrequireamaterialmedium*for their propagation, are called electromagnetic waves.**

  In the case of electromagnetic waves, there are oscillating electric and magnetic fields mutually at right angles and at right angles to the direction of wave propagation.

Example:Light waves, radio waves, infra redwaves, x-rays, γ-rays etc.

☞ Electromagnetic waves can travel through matter and also through vacuum but mechanical waves cannot travel through vacuum.

2. One dimensional, two dimensional and three dimensional waves

Based on the way the waves spread, waves are classified as

- **One-dimensional waves:** Waves which travel along a straight line.

  Example:waves along a stretched string.
- **Two-dimensional waves:** Waves which travel in a plane.

  Example: waves on the still surface of a pond.
- **Three-dimensional waves:** Waves which travel in space and spread in all directions around the source.

  Example: sound waves in air.

3. Longitudinal and transverse waves

Based on the direction of vibration of the particles of a medium with respect to the direction of propagation, waves are classified as

- **Longitudinal waves**

  Waves in which the oscillations of the particles of a medium are along the direction of wave propagation are **called longitudinal waves.**

  Longitudinal waves are formed by a sequence of alternate compressions and rarefactions.

Example: **sound waves.**

- **Transverse waves**

  Waves in which the oscillations of the particles of a medium are perpendicular to the direction of wave propagation are called**transverse waves.**

  A transverse wave consists of a sequence of crests and troughs.

  Example: **waves on the surface of water, waves on a string.**

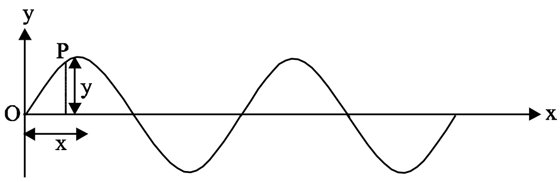
Characteristics of mechanical progressive waves

1. Waves are produced by continuous periodic vibrations generated at a point in a medium.
2. The elastic and inertial property of a medium is responsible for wave propagation.
3. Waves transport energy (and momentum) from the source of vibration (disturbance) away from it.

   However, the particles of matter themselves do not move away. They only perform simple harmonic vibrations about their mean positions.
4. Waves in a homogeneous medium travel with constant velocity at a given temperature.
5. All the vibrating particles along a wave vibrate with the same frequency. For a three dimensional wave, the amplitude decreases as the distance from the source increases.
6. Waves undergoreflection, refraction, diffraction etc.
7. Only transverse waves undergo polarisation.

8. Wave propagation is longitudinal inside solids, liquids and gases. Wave propagation can be transverse on a liquid surface, inside solids and on strings.

Wave equation



Consider a stretched string, which is in equilibrium along x-axis.

Let the free end of the string be subjected to periodic up-down motion. A wave is setup in the string. During the wave motion, the maximum displacement of each particle in the string is the same along a direction perpendicular to the string.

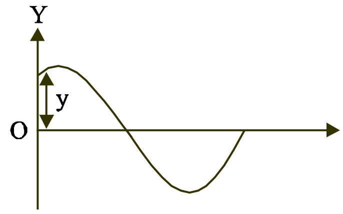
- The displacement y of a particle on the string at an instant of time depends on its x - coordinate.
- The displacement y of a particle at a given value of x depends on time.

Thus, in general, the displacement is, y = f (x, t)

An equation that gives the displacement of a particle in a medium at any point at any instant of time is called a wave equation.

A general form of the wave equation is y = f(x ± v t).

Displacement relation in a wave (Equation for a progressive wave)



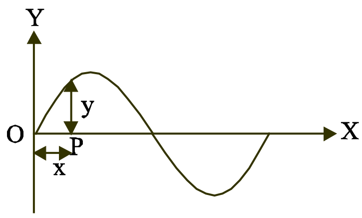
Consider a sinusoidal wave. As the wave travels along the positive x-direction, particles of the medium perform simple harmonic vibrations. Different particles along the wave are in different states of vibration.

Let the displacement of the particle at the origin (x = 0) at an instant be represented by

$y = A\sin\omega t$ ... (1)

where *A* is the amplitude and ω is the angular frequency.

This equation can represent the displacement of the particle at the origin. This equation can be converted into a general equation as follows.



A particle at P exhibits the same state of vibration as that of a particle at the origin after a time lag. The time taken by the disturbance to reach the point P at a distance x from the origin is $\frac{x}{v}$ where, v is the velocity of the wave. The state of motion of the particle at P is same as that of the particle at the origin at a slightly earlier time $= \left(t - \frac{x}{v}\right)$.

i.e., displacement of particle P at a distance x at time t = displacement of a particle at O at time $\left(t - \frac{x}{v}\right)$.

RHS is given by equation (1) on replacing t by $\left(t - \frac{x}{v}\right)$.

We get the equation for the motion of the particle at P as

$$y = A\sin\omega\left(t - \frac{x}{v}\right) \qquad \text{... (2)}$$

Therefore, $y = A\sin(\omega t - kx)$ ... (3)

where $k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$ is called **angularwave number** or **propagation constant.**

Equations (2) and (3) represent a wave travelling along the **positive X-direction**.

In equation (3), (ωt –kx) represents the phase of a particle at distance x from the origin at time t.

Other forms of wave equation

Substituting $\omega = 2\pi f$ and $f = \frac{v}{\lambda}$ in equation (2), we get $y = A\sin\frac{2\pi}{\lambda}(vt - x)$ ... (4)

Substituting $v = \frac{\lambda}{T}$ in equation (4)we get $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ... (5)

The equations for a wave travelling in the **negative X-direction** are

- $y = A\sin\omega\left(t + \frac{x}{v}\right)$
- $y = A\sin(\omega t + kx)$
- $y = A\sin\frac{2\pi}{\lambda}(vt + x)$
- $y = A\sin 2\pi\left[\frac{t}{T} + \frac{x}{\lambda}\right]$

Intensity of a wave

Intensity of a wave is defined as the average energy transported by the wave per unit time across unit area perpendicular to the direction of wave propagation.

Expression for Intensity of a wave

In the case of a sinusoidal mechanical wave of amplitude A, frequency f, propagating with a velocity v in a medium of density ρ, the intensity I is given by,

$$I = 2\pi^2 A^2 f^2 \rho v$$

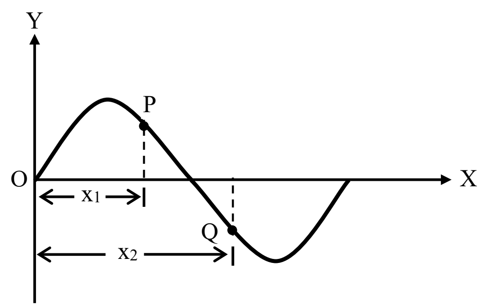
The SI unit of intensity is J s^{-1} m^{-2} or watt per meter square W m^{-2}

- $I \propto A^2$, $I \propto f^2$, $I \propto \rho$, $I \propto v$
- $I \propto \frac{1}{d^2}$ for a point source in an isotropic medium, when the absorption by the medium is negligible.
- $v \propto \sqrt{T}$ and $\rho \propto \frac{1}{T} \Rightarrow I \propto \frac{1}{\sqrt{T}}$ (where T = temperature)
- $I \propto p$ (because $\rho \propto p$ and v is independent of p, p = pressure)

Principle of superposition

When two or more waves of the same nature travel past a point simultaneously, the net disturbance at the point is the vector sum of the disturbances due to the individual waves.

Relation between phase difference and path difference



The equation for a simple harmonic wave travelling in the positive X-direction is $y = A\sin\omega(t - x/v)$ where

$\omega(t - x/v) = \phi$ is the phase of a particle at a distance x from the origin in the positive X-direction.

If x_1 and x_2 are distances of two particles from the origin, then the phase difference

$\phi_1 - \phi_2 = \Delta\phi = \omega(t - x_1/v) - \omega(t - x_2/v)$

i.e., $\Delta\phi = \omega\left[\frac{x_2 - x_1}{v}\right]$ or $\Delta\phi = \omega\left[\frac{\Delta x}{f\lambda}\right] = 2\pi f\left[\frac{\Delta x}{f\lambda}\right] = 2\pi\left[\frac{\Delta x}{\lambda}\right]$

where Δx = path difference.

Thus, $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$ Hence, $\boxed{\text{phase difference} = \frac{2\pi}{\lambda}\ (\text{path difference})}$

Illustrations

1. Transverse waves can propagate

(A) both in a gas and a metal (B) in a gas but not in a metal

(C) not in a gas but in a metal (D) neither in a gas nor in a metal

Ans (C)

2. The equation of a plane progressive wave is given by y = 2sin (5πt – 0.5πx). The speed of the wave is [x and y are in metres and t is in seconds]

(A) 10 m s^{-1} (B) 5 m s^{-1} (C) 0.5 m s^{-1} (D) 2.5 m s^{-1}

Ans(A)

Compare the equation with the standard form y = A sin (ωt – kx),

ω = 5π rad/s, k = 0.5π rad/m

Wave speed $v = \frac{\omega}{k} = \frac{5\pi}{0.5\pi} = 10\ m\ s^{-1}$

3. The equation of a progressive wave is given by y = 2sin(πt – 0.5πx) where x and y are in metres and t is in seconds. The speed of a particle at t = 1 s and x = 1 m is

(A) 2 m s^{-1} (B) zero (C) 4 m s^{-1} (D) 0.5 m s^{-1}

Ans (B)

Comparing the equation with the standard form, y = A sin (ωt – kx)

A = 2 m, ω = π rad/s

At x = 1 m and t = 1 s, y = 2sin (π – 0.5π) = 2sin 0.5π = 2 m

The particle speed is given by $v_P = \omega\sqrt{A^2 - y^2}$

$$= \pi\sqrt{2^2 - 2^2} = 0$$

4. A particle executing SHM has an acceleration of 0.5 m s^{-2} for a displacement of 2 m. Its period is

(A) 1.256 s (B) 12.56 s (C) 125.6 s (D) 0.1256 s

Ans (B)

$a = 0.5\ ms^{-2}$ at y = 2 m

Acceleration of a particle executing SHM is $a = \omega^2 y = \left(\frac{2\pi}{T}\right)^2 y \Rightarrow T^2 = \frac{4\pi^2 y}{a}$

$$= \frac{4\pi^2 \times 2}{0.5} = 16\pi^2$$

$\Rightarrow T = 4\pi = 4 \times 3.14 = 12.56$ s

5. The equation of progressive wave is given by $y = a \sin \pi\left(\frac{t}{2} - \frac{x}{2}\right)$, where 't' is in seconds and x is in metre. The distance through which the wave moves in 8 seconds is in (metre)

(A) 8 m (B) 16 m (C) 4 m (D) none

Ans(A)

$y = a \sin \pi\left(\frac{t}{2} - \frac{x}{2}\right)$ is given equation.

The standard wave equation, $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

The given equation can be written as,

$y = a\sin 2\pi\left[\frac{t}{4} - \frac{x}{4}\right]$ comparing.

$T = 4,\ \lambda = 4\ \therefore$ distance = velocity × time

$\frac{\lambda}{T} \times t$

$\frac{4}{4} \times 8 = 8\ m$

6. The equation of a transverse wave travelling along positive x axis with amplitude 0.1 m velocity 360 ms^{-1} and wavelength 60 m can be written as,

(A) $y = 0.1 \sin 2\pi\left(6t - \frac{x}{60}\right)$

(B) $y = 0.1 \sin 2\pi\left(6t + \frac{x}{60}\right)$

(C) $y = 0.1 \sin \pi\left(6t + \frac{x}{60}\right)$

(D) $y = 0.1 \sin \pi\left(6t - \frac{x}{60}\right)$

Ans(D)

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

$$= 0.1\sin\frac{2\pi}{60}(360 \times t - x) = 0.1\sin 2\pi\left(6t - \frac{x}{60}\right)$$

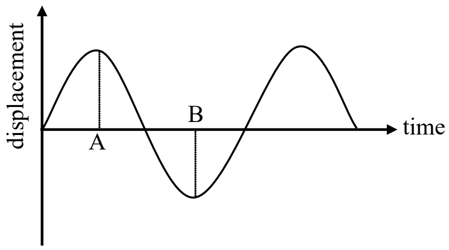
7. In the diagram shown, the distance travelled by the wave in the time interval AB is

(A) wavelength

(B) $\frac{\text{wave length}}{2}$

(C) $\frac{\text{period}}{2}$

(D) 2 × amplitude



Ans (B)

Distance between a crest (A) and the nearest trough (B) = $\frac{\lambda}{2}$

8. A source of frequency f sends waves of wavelength λ travelling at speed v in a medium. The wavelength and the speed of a wave of frequency 4f in the same medium are respectively

(A) 4λ and v

(B) 4λ and 4v

(C) $\frac{\lambda}{4}$ and v

(D) $\frac{\lambda}{4}$ and $\frac{v}{4}$

Ans (C)

Speed of a wave in a medium is independent of frequency. Therefore, the speed v remains the same. Speed $v = f\lambda \Rightarrow \lambda \propto \frac{1}{f}$. Therefore $\lambda^1 = \frac{1}{f^1} = \frac{1}{4f} = \frac{\lambda}{4}$.

9. The displacement of a progressive wave is describe by the equation y = 5sin(60t + 2x) where x and y are in metres and t in seconds. This wave has

(A) wavelengths equal to 2πmetres

(B) period of oscillation equal to $\frac{\pi}{15}$sec

(C) velocity equal to 30 m/s along the negative x direction

(D) frequency equal to $\frac{15}{\pi}$Hz

Ans(C)

Equation of progressive wave is, $y = A \sin(\omega t - kx)$

Given equation $y = 5\sin(60t + 2x) = 5\sin\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x\right)$

$\frac{2\pi}{T} = 60 \quad \therefore \frac{1}{T} = f = \frac{30}{\pi}$ and $\frac{2\pi}{\lambda} = 2$ and $\lambda = \pi$

$v = f\lambda = \frac{30}{\pi} \times \pi = 30 \text{ ms}^{-1}$ in –ve X direction

10. The equation for a plane progressive wave is given by, $y = 5\cos \pi\left(200t - \frac{x}{150}\right)$, where x and y are in cm and t is in second. The velocity of the wave is

(A) 2 m s^{-1} (B) 200 m s^{-1} (C) 300 m s^{-1} (D) 150 m s^{-1}

Ans(C)

Comparing with the standard equation

$y = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$,

we get, A = 5 cm, $\frac{2\pi}{T} = 200\pi \Rightarrow T = \frac{1}{100}$

$\Rightarrow f = 100$ Hz and $\frac{2\pi}{\lambda} = \frac{\pi}{150} \Rightarrow \lambda = 300 \text{ cm} = 3 \text{ m}$

$\therefore v = 300 \text{ m s}^{-1}$

11. Sound waves of wavelength λ travelling with velocity v in a medium enter another medium in which their velocity is 4v. The wavelength in the second medium is

(A) 4λ (B) λ (C) $\frac{\lambda}{4}$ (D) 16λ

Ans(A)

When a wave travels across a medium, the frequency of the waves remains constant.

$\therefore f_1 = f_2$

$\left(\frac{v}{\lambda}\right) = \frac{(4v)}{\lambda_1} \therefore \lambda_1 = 4\lambda$

12. When a wave is reflected from a denser medium, the change in phase in radians is

(A) 0 (B) π (C) 2π (D) 3π

Ans(B)

13. A sound wave travelling in the +x direction has an amplitude 0.01 m, frequency 125 Hz and velocity of propagation 375 m s^{-1}. Find the displacement, velocity and acceleration of a particle in the medium situated 1.5 m from the origin at t = 5 seconds.

Solution

$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$, $A = 0.01$ m, $v = 375 \text{ m s}^{-1}$, $f = \frac{1}{T} = 125$ Hz, x = 1.5 m

$\lambda = \frac{v}{f} = 3$ m, t = 5 s

$y = 0.01\sin 2\pi\left(125 \times 5 - \frac{1.5}{3}\right) = 0.01\sin 2\pi[625 - 0.5] = 0.01\sin 2\pi[624.5]$

$= 0{\cdot}01\sin 1249\pi = 0.01\sin\pi = 0$

Particle velocity, $\frac{dy}{dt} = A\frac{2\pi}{T}\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 0.01 \times 2\pi \times 125 \times 1 = 2.5\pi \text{ m s}^{-1}$

Particle acceleration, $\frac{d^2y}{dt^2} = A\left(\frac{2\pi}{T}\right)^2 \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 0$

14. A tuning fork is at rest. When it is set into oscillations and allowed to fall freely, eight complete oscillations are counted in falling through 10 cm. What is the frequency of the fork?

Solution

Time taken to fall by the tuning fork is given by, $s = \frac{1}{2}gt^2$

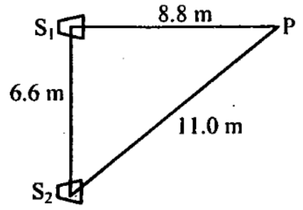
$0.1 = \frac{1}{2} \times 9.8 \times t^2 \Rightarrow t^2 = \frac{0.1}{4.9}$

$t = \frac{1}{7}s$

In $\frac{1}{7}$ s, it makes 8 vibrations.

$\therefore$ in 1 s, 8×7 vibrations $\quad \therefore f = 56$ Hz.

15. Two speakers that are in synchronization, are connected to a sine wave source. Waves of 2.2 m wavelength travel to point P from the speakers. The phase difference, $\Delta\phi$, between the waves from S_2 and S_1 when they arrive at point P is



(A) π (B) 2π

(C) 6π (D) 8π

Ans(B)

$$\Delta\phi = \frac{2\pi}{\lambda}(p.d) = \frac{2\pi}{2.2}(2.2) = 2\pi$$

16. From a point source, if amplitude of waves at a distance r is A, its amplitude at a distance 2r will be

(A) A (B) 2A (C) $\frac{A}{2}$ (D) $\frac{A}{4}$

Ans(C)

$$I \alpha \frac{1}{r^2} \text{ and } I \,\alpha\, A^2 \Rightarrow A \,\alpha\, \frac{1}{r}$$

17. The displacement of the particles of the medium when wave is propagating through the medium is given by y = A cos (ax + bt) where A, a and b are positive constants The wave is reflected by an obstacle at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. The wavelength and frequency of the incident wave are

(A) $\lambda = \frac{\pi}{a}$ and $f = \frac{b}{2\pi}$ (B) $\lambda = \frac{2\pi}{a}$ and $f = \frac{b}{2\pi}$

(C) $\lambda = \frac{2\pi}{a}$ and $f = \frac{b}{\pi}$ (D) $\lambda = \frac{\pi}{a}$ and $f = \frac{b}{\pi}$

Ans(B)

$$y = A\cos(ax + bt) = A\cos(kx + \omega t) \Rightarrow k = a$$

$$\frac{2\pi}{\lambda} = a \quad \therefore \lambda = \frac{2\pi}{a} \quad \omega = b,\ 2\pi f = b \quad \therefore f = \frac{b}{2\pi}$$

18. The displacement of the particles of the medium when wave is propagating through the medium is given by y = A cos(ax + bt) where A, a and b are positive constants. The wave is reflected by an obstacle at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. The equation for the reflected wave is

(A) + 0.8A cos (bt – ax) (B) + 4A cos (bt – ax)

(C)– 0.8A cos (bt – ax) (D)– 4A cos (bt – ax)

Ans(C)

When wave gets reflected from an obstacle, phase of the wave changes by π radians.

$$I_r = 0.64\ I_i \Rightarrow A_r^2 = 0.64\ A^2 \text{ or } A_r = 0.8\ A$$

$\therefore$ Reflected wave $y_2 = -0.8\,A\cos(bt - ax)$

19. The displacement of the particles of the medium when wave is propagating through the medium is given by y = A cos(ax + bt) where A, a and b are positive constants The wave is reflected by an obstacle at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. In the resultant wave formed after reflection, the maximum and minimum values of the particle speeds in the medium are

(A) $v_{max} = 1.8$ Ab and $v_{min} = 0.2$ Ab
(B) $v_{max} = 1.6$ Ab and $v_{min} = 0.1$ Ab
(C) $v_{max} = 1.4$ Ab and $v_{min} = 0.1$ Ab
(D) $v_{max} = 1.2$ Ab and $v_{min} = 0.2$ Ab

Ans(A)

Particle speed due to incident wave is $(v_p)_1 = \frac{dy_1}{dt} = -Ab\,\sin(ax + bt)$

$(v_p)_{max} = Ab$

Particle speed due to reflected wave is $(v_p)_2 = \frac{dy_2}{dt} = 0.8\,Ab\sin(bt - ax)$

$(v_p)_{max} = 0.8\,Ab$

$\therefore$ maximum value of particle speed is $(v_p)_{max} = v_{p_1} + v_{p_2} = 1.8\,Ab$

and minimum value of particle speed is $(v_p)_{min} = Ab - 0.8\,Ab = 0.2\,Ab$

20. The equation of a progressive wave is given by $y = 0.2\sin 2\pi\left(60t - \frac{x}{5}\right)$, where x and y are in metre and t is in second. The phase difference at any point between two instants $\frac{1}{120}$s and $\frac{1}{40}$s is

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) π
(D) 2π

Ans(D)

Comparing the given equation with the standard wave equation

$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ we find that $T = \frac{1}{60}$s

Phase difference $\Delta\phi = \frac{2\pi}{T}(\Delta t) = 2\pi\left(\frac{\frac{1}{40} - \frac{1}{120}}{\frac{1}{60}}\right) = 2\pi$

21. The phase difference between two points separated by 0.8 in wave of frequency 120 Hz is 0.5π. Thewave velocity is

(A) 144 m s^{-1}
(B) 256 m s^{-1}
(C) 384 m s^{-1}
(D) 720 m s^{-1}

Ans (C)

22. The equation of a travelling wave is y = 60cos (1800t – 6x), where y is in microns, t in seconds and x in metres. The ratio of maximum particle velocity to velocity of wave propagation is

(A) 3.6
(B) 3.6×10^{-6}
(C) 3.6×10^{-11}
(D) 3.6×10^{-4}

Ans(D)

23. A string of length L is stretched by L/20 and the speed of transverse waves along it is v. The speed of wave when it is stretched by L/10 will be (assume that Hooke's law is applicable)

(A) 2v
(B) $\frac{v}{\sqrt{2}}$
(C) $\sqrt{2}$ v
(D) 4v

Ans(C)

24. A sound wave y = A sin(ωt –kx) is propagating through a medium of density ρ. The sound energy per unit volume is

(A) $\frac{1}{2}\rho A^2\omega^2$
(B) $2\rho A^2\omega^2$
(C) $\rho A^2\omega^2$
(D) $4\rho A^2\omega^2$

Ans(A)

Intensity $I = 2\pi^2 f^2 A^2 \rho \upsilon = \frac{1}{2}(4\pi^2 f^2) A^2 \rho \upsilon$

$\Rightarrow I = \frac{1}{2}\omega^2 A^2 \rho \upsilon$

The energy density $u_g = \frac{\text{Energy}}{\text{volume}} = \frac{u_E}{V} \Rightarrow u_E = \frac{1}{2}\rho\omega^2 A^2$.

25. In the interference of waves from two sources of intensities I_0 and $4I_0$. The intensity at a point where thephase difference is $\pi$is

(A) $5I_0$ (B) $3I_0$ (C) $2I_0$ (D) I_0

Ans(D)

In interference, amplitudes get added vectorially. Here amplitudes are proportional to $\sqrt{I_0}$ and $2\sqrt{I_0}$ because intensity $\propto$ amplitude^2

$\therefore$ Resultant amplitude $\propto \sqrt{I_0 + 4I_0 + 2\sqrt{I_0}\,2\sqrt{I_0}\cos\pi} = \sqrt{I_0}$

$\therefore$ Intensity = I_0

Sound

Sound is a form of energy that produces the sensation of hearing. Sound requires a material medium for its propagation. Sound propagates as a longitudinal wave. The elastic and inertial properties of a medium define the speed of sound in that medium. Sound can propagate in solids, liquids and gases. Vibrating bodies surrounded by a medium produce sound. Human ear is sensitive to sound waves whose **frequencies** lie between 20 Hz and 20,000 Hz. Sound waves of **frequencies** lower than 20 Hz are called **infrasonics.** Sound waves of frequencies higher than 20,000 Hz are called **ultrasonics.**

☞ 'Supersonic' means, faster than sound and 'subsonic' means slower than sound. These words are used with respect to the velocity of a body as compared to the velocity of sound.

The branch of physics that deals with the study of sound is called **Acoustics**.

Waves generated by vibrating bodies in air, produce a sequence of compressions and rarefactions. Thus, sound is a longitudinal wave comprising of compressions and rarefactions.

☞ In a medium, if waves of all frequencies travel with the same speed, the medium is called a "non-dispersive medium". If waves of different frequencies travel with different speeds, the medium is called a dispersive medium.

Properties of sound waves

1. **Sound waves require a material medium for propagation.**
2. **Sound waves are mechanical waves, longitudinal in nature comprising of a sequence of alternate compressions and rarefactions.**
3. Sound waves travel at a constant speed at a given temperature in a homogeneous medium.
4. Sound waves of all frequencies travel with the same speed in air (air is non dispersive to sound).

5. When a sound wave travels from one medium to another, its velocity and wavelength change but its frequency remains unaltered.
6. Sound waves transport energy.
7. Sound waves undergo reflection, refraction etc, but not polarization since the waves are longitudinal.
8. Sound waves bend around obstacles (diffraction).
9. Sound waves exhibit interference (beats).
10. Sound waves incident on a surface set the surface into vibrations.

Velocity of sound in gases

The speed of a mechanical wave in a medium depends on the elastic and inertial properties of the medium.

The **velocity** (v) of a mechanical wave in a medium of **density** (ρ)and **bulk modulus** (K) is given by

$$v = \sqrt{\frac{K}{\rho}} \qquad \text{... (1)}$$

Newton's formula

Newton assumed that the temperature of different layers of air in the regions of compressions and rarefactions remains constant during wave propagation. Thus, sound propagation takes place under isothermal conditions. For an isothermal process, we have pV = constant. Using this, it can be shown that under isothermal conditions, the **bulk modulus** (K) **is equal to the pressure** (p) of the gas. i.e., K = p.

Thus, equation (1) can be written as $v = \sqrt{\frac{p}{\rho}}$ ... (2)

Need for correction

At STP, $p = \rho_{Hg} gh = (13600 \times 9.8 \times 0.76)\ N\ m^{-2}$ and density of air= $1.293\ kg\ m^{-3}$

Substituting these values in equation (2) we get, v= $280\ m\,s^{-1}$

The experimental value of the velocity of sound in air at STP is $331.3\ m\,s^{-1}$. Thus, the value obtained using Newton's formula does not agree with the experimental value and hence, Newton's formula was discarded.

Newton-Laplace formula

Laplace suggested a correction to Newton's formula. He assumed that the condition prevailing in the compressions and rarefactions is **adiabatic** and **not isothermal**. This is because, the vibrations of the layers of air are so rapid that there is hardly any time for heat transfer between layers and also air is a bad conductor of heat. Under adiabatic conditions, we have pV^{γ}= constant. Using this, it can be shown that the bulk modulus $K = \gamma\, p$.

Hence, $v = \sqrt{\frac{\gamma\, p}{\rho}}$ ··· (3)

$\gamma\, p$ is called **adiabatic elasticity**, and γ = ratio of molar specific heats of a gas $(\gamma = C_P/C_V)$.

The introduction of γ into Newton's formula is called **Laplace's correction.**

Equation (3) is called **Newton-Laplace formula**.

For air, γ = 1.41. Using this value, we get $v = \sqrt{\frac{1.41 \times 13600 \times 9.8 \times 0.76}{1.293}} = 332\ m\ s^{-1}$

This value obtained for v agrees with the experimental value.

Newton's formula	Newton-Laplace formula
$v = \sqrt{\frac{p}{\rho}}$; p = isothermal elasticity, ρ = density of the gas	$v = \sqrt{\frac{\gamma p}{\rho}}$; γp = adiabatic elasticity, ρ = density of the gas, γ = ratio of specific heats.

Velocity at STP calculated from this formula gives a value of 280 m s^{-1}, which doesn't agree with experimentally determined value.	Velocity at STP calculated from this formula gives a value of 330 m s^{-1}, which agrees with experimentally determined value.

Factors affecting the velocity of sound in air

- **Effect of pressure**

  When the pressure of a gas increases at constant temperature, its volume decreases (Boyle's law). When the volume decreases, the density increases such that $\frac{p}{\rho}$ = constant.

  Hence, it can be seen from equation (3) that "**the velocity of sound in a gas is independent of pressure, provided the temperature is constant".**

- Effect of temperature

  Newton-Laplace formula is $v = \sqrt{\frac{\gamma p}{\rho}}$ .Density $\rho = \text{mass}/\text{volume} = m/V$.

  $\therefore$ For a given mass of a gas, volume $V \propto \frac{1}{\rho}$

  From Charle's law, we have, at constant pressure $V \propto T$ $\quad \therefore \frac{1}{\rho} \propto T$

  At constant pressure, if T is varied, we have $v \propto \sqrt{\frac{1}{\rho}}$ or $v \propto \sqrt{T}$ $\therefore$ $v \boxed{\propto \sqrt{T}}$

  where, T is the temperature of the gas on the absolute scale.

  Hence, velocity of sound in a gas varies directly as the square root of its absolute temperature.

- **Effect of humidity**

  Humidity is the measure of the amount of **water vapour present in the atmosphere**. The density of water vapour (is about 5/8 times that of dry air) is lower than that of dry air. Thus, presence of humidity decreases the density of air. Hence, with **humidity, the velocity of sound in air increases.** i.e., sound travels faster in humid air than in dry air.

- **Effect of wind**

  If w is the wind velocity opposite to the direction of sound propagation, then the effective velocity of sound is (v – w). If wind blows in the direction of sound propagation, the effective velocity of sound is (v + w).

☞

- Velocity of sound in a gas varies directly as the square root of its absolute temperature. Hence, if v_1 and v_2 are the velocities of sound at temperatures T_1 and T_2, then $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$
- Density $\rho = \frac{m}{V}$ $\therefore$ $v = \sqrt{\frac{\gamma p V}{m}} = \sqrt{\frac{\gamma nRT}{m}} = \sqrt{\frac{\gamma RT}{M}}$ $\{\because pV = nRT \text{ and } \frac{m}{n} = M = \text{molecular weight}\}$
- If v_0 and v are the velocities of sound in air at 0 °C and t °C respectively, then we have

$T_1 = 273$ K and $T_2 = (273 + t)$ K

Since velocity of sound in a gas varies directly as the square root of its absolute temperature, we have

$$\frac{v}{v_0} = \sqrt{\frac{273+t}{273}} = \sqrt{1+\frac{t}{273}} = \left[1+\frac{t}{273}\right]^{1/2}. \text{ Hence, } v = v_0\sqrt{1+\frac{t}{273}}$$

$(1 + x)^n \approx 1 + nx$ (if $x \ll 1$) is called binomial approximation.

For small values of t, $\left(\frac{t}{273}\right)$ is small and hence, using binomial approximation we get.

$v = v_0\left[1+\dfrac{t}{2\times 273}\right]$ That is, $v = v_0\left[1+\dfrac{t}{546}\right]$, also $v_0 = v\left[1-\dfrac{t}{546}\right]$

Since, $v - v_0 = v_0\left[\dfrac{t}{546}\right]$ and (1/546) ≈ 0.0018, the velocity of sound increases approximately by 0.18 % of v_0 or 0.6 m s^{-1} for every degree rise in temperature.

Illustrations

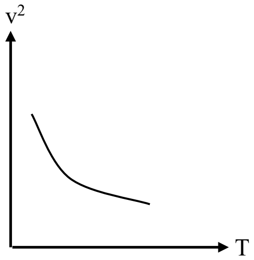
1. Sound travels faster in

(A) water (B) air (C) steel (D) wood

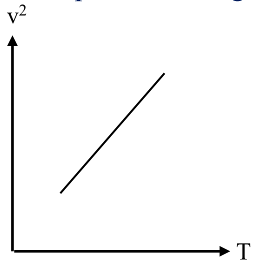
Ans (C)

2. The variation of the square of speed of sound (v^2) with absolute temperature of a gas (T) is shown correctly in

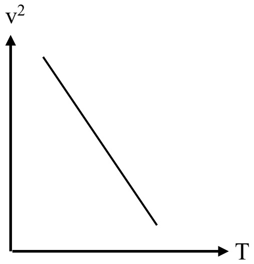
(A)



(B)



(C)



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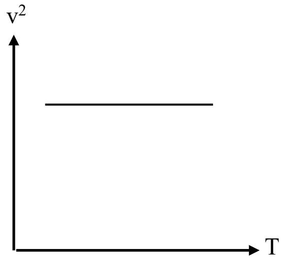


Ans (B)

Since $v \propto \sqrt{T}$, we get $v^2 \propto T$ which is represented in figure (B).

3. Velocity of sound is ______ on a rainy day than on a sunny day.

(A) faster (B) slower (C) the same (D) faster or slower

Ans (A)

4. With decrease in water vapour content in air, velocity of sound

(A) increases (B) decreases (C) remains constant (D) cannot say

Ans (B)

$v = \sqrt{\dfrac{\gamma p}{\rho}}$

With the decrease in water vapour, the density increases and thus, the velocity of sound in air decreases.

5. Under identical conditions of pressure and density, the speed of sound is highest in

(A) monatomic gas (B) diatomic gas (C) triatomic gas (D) polyatomic gas

Ans (A)

As $v = \sqrt{\dfrac{\gamma p}{\rho}}$ and γ is highest for a monatomic gas, the velocity of sound is highest in a monatomic gas.

6. The velocity of sound in vacuum is

(A) zero (B) 10 m s^{-1}

(C) 332 m s^{-1} (D) cannot be determined

Ans (A)

Sound is a mechanical wave requiring a material medium for its propagation.

7. When a sound wave travels through a medium, the temperature of the medium,

(A) changes (B) remains constant

(C) changes or remain constant (D) data isinsufficient

Ans (A)

The sound propagation in a medium takes place adiabatically involving a temperature change.

8. The speed of sound in solids is maximum though their density is large because of

(A) high coefficient of elasticity (B) low coefficient of elasticity

(C) high coefficient of expansion (D) high coefficient of thermal conductivity

Ans (A)

9. A sound wave of wavelength 90 cm in glass is refracted into air. If the velocity of sound in glass is 5400 m s^{-1}, the wavelength of the wave in air is

(A) 55 cm (B) 5.5 cm (C) 55 cm (D) 5.5 m

Ans(B)

Velocity = frequency × wavelength

In glass: 5400 × 100 = n × 90 or $n = \frac{5400 \times 100}{90}$ Hz

In air: $\frac{330 \times 100}{\lambda} = n = \frac{5400 \times 100}{90}$ $\therefore \lambda = \frac{330 \times 100 \times 90}{5400 \times 100} = 5.5$ cm.

10. The increase in the speed of sound in air for every degree Celsius rise in temperature is (velocity of sound at 0 °C is 330 m s^{-1})

(A) 6 m s^{-1} (B) 0.6 m s^{-1} (C) 0.12 m s^{-1} (D) 1.2 m s^{-1}

Ans (B)

We know that $v_0 \propto \sqrt{273}$ and $v_t \propto \sqrt{273+1} = \sqrt{274}$

or $\frac{v_0}{v_t} = \sqrt{\frac{273}{274}}$ $\therefore v_t = v_0 \sqrt{\frac{274}{273}}$

Difference in speed $\delta v = v_t - v_0 = v_0\left[\left(\frac{274}{273}\right)^{1/2} - 1\right] = 0.6$ m s^{-1}

11. Sound travels fastest in

(A) water (B) air (C) steel (D) wood

Ans (C)

12. Velocity of sound is ______ on a rainy day than on a sunny day.

(A) faster (B) slower (C) the same (D) faster or slower

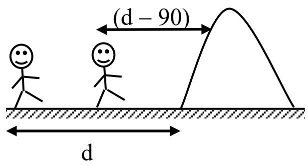
Ans (A)

13. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. Then, he moves nearer to the mountain by 90 m and finds that the echo is again not heard when the drumming rate becomes 60 per minute. Calculate,

(a) the distance between the mountain and the initial position of the man.

(b) the velocity of sound.

Solution



Time taken by echo to reach the person $t_1 = \frac{2d}{v}$

Interval between two successive beats is $\frac{60}{40} = 1.5$ s

Echo is not heard, if successive beating overlaps with the reflected sound

$\therefore 2\left(\frac{d}{v}\right) = 1.5$ … (1)

When person moves towards the mountain

$t_2 = \frac{2(d-90)}{v}$, now interval is $\frac{60}{60} = 1$ s

$\Rightarrow \frac{2d-180}{v} = 1$ … (2)

From (1) and (2) d = 270 m and v = 360 m s^{-1}.

14. Velocity of sound at 0 °C 332 m s^{-1}. What are the wavelengths corresponding to the sounds of frequencies 161 Hz and 311 Hz, at 16 °C?

Solution

$v_t = v_0(1 + \alpha t) \Rightarrow v_{16} = 351.5$ m s^{-1}

$\lambda = \frac{v}{f} \Rightarrow 2.18$ m

15. An engine approaches a wall with a constant speed. When it is at a distance of 0.9 km, it blows a whistle, whose echo is heard by the driver after 5 s. If the speed of sound in air in 330 m s^{-1}, calculate the speed of the engine.

Solution

$t = t_1 + t_2$

$5 = \frac{900}{330} + t_2 \Rightarrow t_2 = 2.27$

Distance travelled by echo in 2.27 s = 330 × 2.27 ≈ 750 m

Distance travelled by engine = 900 – 750 = 150

$\therefore v_s = \frac{150}{5} = 30$ m s^{-1}

16. Calculate the time taken by sound waves to travel a distance of l between the points A and B if the air temperature between them varies linearly from T_1 to T_2. The velocity of sound in air is given by $v = \alpha\sqrt{T}$ where α is a constant.

Solution

$v_A = \alpha\sqrt{T_1}$, $v_B = \alpha\sqrt{T_2}$

$$\text{time} = \frac{d}{v_{av}} = \frac{l}{\alpha\frac{(\sqrt{T_1}+\sqrt{T_2})}{2}} = \frac{2l}{\alpha(\sqrt{T_1}+\sqrt{T_2})}$$

17. The speed of sound in solids is maximum though their density is large because of

(A) high coefficient of elasticity (B) low coefficient of elasticity

(C) high coefficient of expansion (D) high coefficient of thermal conductivity

Ans (A)

18. A sound wave of wavelength 90 cm in glass is refracted into air. If the velocity of sound in glass is 5400 m s^{-1}, the wavelength of the wave in air is

(A) 55 cm (B) 5.5 cm (C) 55 cm (D) 5.5 m

Ans(B)

Velocity = frequency × wavelength

In glass: $5400 \times 100 = n \times 90$ or $n = \dfrac{5400 \times 100}{90}$ Hz

In air: $\dfrac{330 \times 100}{\lambda} = n = \dfrac{5400 \times 100}{90}$ $\therefore \lambda = \dfrac{330 \times 100 \times 90}{5400 \times 100} = 5.5$ cm

Stationary Waves and Beats

Consider two progressive waves of the same amplitude and wavelength travelling along a line in opposite directions.

Let the waves be represented by

$y_1 = A \sin(\omega t - kx)$, $y_2 = A \sin(\omega t + kx)$

The waves interfere to form a stationary wave. The equation for the resultant stationary wave is obtained using the principle of superposition. If y is the resultant displacement, then

$y = y_1 + y_2 = A \sin(\omega t - kx) + A \sin(\omega t + kx)$

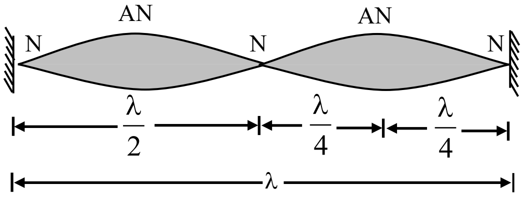
We can show that the above relation reduces to $y = 2A \cos kx \sin \omega t$ … (1)

The above equation represents a standing wave.

A stationary wave or a standing wave is one which is formed when two waves of equal amplitude and wavelength travel along a line in opposite directions and superimpose.

☞ In equation (1) the resultant amplitude is 2A cos kx. Note that this is independent of time, but, varies with x.
If the waves are represented by cosine functions, we get $y = 2A \sin kx \sin \omega t$.
To produce the superposition of a pair of identical waves, the given wave is made to get reflected from a surface and the reflected wave is superposed on the incident wave.

Stationary waves have very interesting properties. When stationary waves are formed, points of no vibration and points of maximum amplitude of vibration are formed. Points of no vibration are called **nodes** and points of maximum amplitude of vibration are called **antinodes**. Further, all the vibrating particles between two consecutive nodes will be in phase. Particles in the adjacent loops will be out of phase by 180°.



N = node
AN = Antinode

☞ Nodes are the points in a stationary wave, where the particles do not vibrate. Antinodes are points in a stationary wave, where the particles vibrate with maximum amplitude.

The distance between two consecutive nodes or antinodes is (λ/2). The distance between a node and the next antinode is (λ/4). The stationary wave pattern between two nodes is called a **segment** or a **loop**.

Characteristics of stationary wave

1. A stationary wave is localized, which exists between two fixed points.
2. It doesn't transport energy.
3. It has alternate nodes and antinodes which are equally spaced, with a separation of (λ/2) between two consecutive nodes or antinodes.

4. Particle at nodes do not vibrate at all while the particles at antinodes vibrate with maximum amplitude.
5. All the particles cross the mean position simultaneously twice during each cycle. But all the particles do not cross the mean position with the same speed.
6. All the particles of the medium in a segment (between two consecutive nodes) vibrate in phase and any two particles in adjacent segments vibrate in opposite phase (with a phase difference of π radian).

Stationary waves in air columns (pipes)

Stationary longitudinal waves can be set up in air columns such as pipes. A longitudinal wave entering a pipe at one end gets reflected at the other end. The incident and the reflected waves travelling in opposite directions superpose to form a stationary wave inside the pipe (The displacement nodes and antinodes are often called the pressure antinodes and the pressure nodes respectively).

Pipes are classified into two types

- **Open pipe:** A pipe open at both ends.
- **Closed pipe:**A pipe closed at one end.

Modes of vibration in a closed pipe

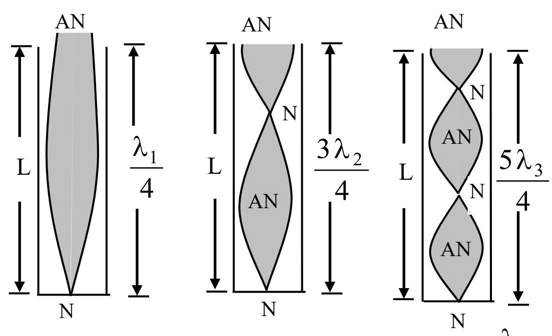
An air column trapped inside a closed pipe vibrates such that the open end always has an antinode and the closed end has a node. This is because the air molecules at the closed end are not free whereas those at the open end have maximum freedom to vibrate.

The frequency of vibration depends on

- **length of the pipe**
- **mode of vibration**

☞	In the figures, the waves are represented graphically. The sound waves are longitudinal.

If the air column vibrates such that the entire air column consists of a single node and a single antinode as shown in figure (1),the mode of vibration is the simplest and is called the **fundamental mode** and the **frequency of vibration is minimum.** This frequency is called the **fundamental frequency.**



(1) (2) (3)

If λ_1 is the **wavelength** and L is the **length** of the pipe, then $L = \frac{\lambda_1}{4}$ or $\lambda_1 = 4L$. If v is the **speed of sound** and f_1 is the **fundamental frequency**,then $f_1 = \frac{v}{\lambda_1}$ i.e. $\boxed{f_1 = \frac{v}{4L}}$ ...(1)

In the next mode of vibration, two nodes and two antinodes are formed. If $\lambda_2$is the wavelength, then from the figure (2), $L = \frac{3\lambda_2}{4}$ or $\lambda_2 = \frac{4L}{3}$

The corresponding frequency $f_2 = \frac{v}{\lambda_2} = \frac{v}{(4L/3)}$ $\boxed{\therefore f_2 = 3\left[\frac{v}{4L}\right] = 3f_1}$ ...(2)

Similarly, in the next higher mode of vibration, three nodes and three antinodes are formed.

From figure (3), $L = \frac{5\lambda_3}{4}$ or $\lambda_3 = \frac{4L}{5}$

Frequency $f_3 = \frac{v}{\lambda_3} = \frac{v}{(4L/5)}$ $\boxed{\therefore f_3 = 5\left[\frac{v}{4L}\right] = 5f_1}$ ...(3)

From equations [1], [2] and [3]it follows that

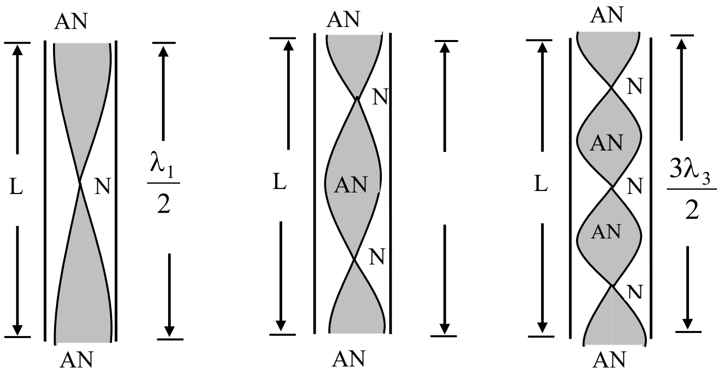
$\boxed{f_1 : f_2 : f_3 \ldots\ldots = 1 : 3 : 5 : \ldots\ldots}$

Hence, **the overtones in a closed pipe are odd multiples of the fundamental**.

☞ The minimum possible frequency is called the fundamental. The integral multiples of the fundamental are called harmonics. The fundamental frequency is called the first harmonic.The higher frequencies are called the overtones. Thus, for a closed pipe, overtones are odd harmonics (the first overtone is the third harmonic; the second overtone is the fifth harmonic etc.)

Modes of vibration in an open pipe

An air column trapped inside an open pipe vibrates such that both the open ends always have antinodes. This is because the air molecules at the open end have maximum freedom to vibrate.



The frequency of vibration depends on

- **length of the pipe**
- **mode of vibration**

The air column vibrates such that the entire air column consists of a single node and two antinodes as shown in figure (1), the mode of vibration is the simplest and is called the **fundamental mode** and the **frequency of vibration is minimum.** This frequency is called the **fundamental frequency.**

If λ_1 is the **wavelength** and **L** is the **length** of the pipe, then $L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$. If v is the **speed of sound** and f_1 is the **fundamental frequency,** then $f_1 = \frac{v}{\lambda_1}$ i.e., $\boxed{f_1 = \frac{v}{2L}}$ ...(1)

In the next mode of vibration, two nodes and three antinodes are formed. If λ_2 is the wavelength, then from figure (2), $L = \lambda_2$.

The corresponding frequency $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$ $\boxed{\therefore f_2 = 2\left[\frac{v}{2L}\right] = 2f_1}$ ...(2)

Similarly, in the next higher mode of vibration, three nodes and four antinodes are formed.

From figure (3), $L = \frac{3\lambda_3}{2}$ or $\lambda_3 = \frac{2L}{3}$

Frequency $f_3 = \frac{v}{\lambda_3} = \frac{v}{\left(\frac{2L}{3}\right)}$ $\boxed{\therefore f_3 = 3\left(\frac{v}{2L}\right) = 3f_1}$ ...(3)

From equations [1], [2] and [3] it follows that $\boxed{f_1 : f_2 : f_3 : \ldots\ldots = 1 : 2 : 3 : \ldots\ldots}$

Hence, **the overtones in an open pipe are integral multiples of the fundamental. i.e., all harmonics are present**.

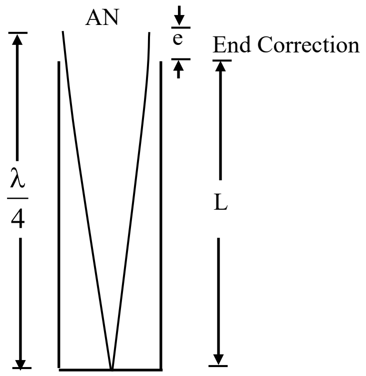
☞ n^{th} overtone of vibrations of air column in an open pipe $f_n = nf_1$; f_1 = fundamental frequency

$$\frac{f_{n\ closed}}{f_{n\ open}} = \frac{(2n+1)f_{1\ closed}}{nf_{1\ open}} = \frac{(2n+1)(v/4L)}{n(v/2L)} = \frac{2n+1}{2n} = 1 + \frac{1}{2n}.$$

End correction

The antinode at the open end of a pipe is not formed exactly at the end of the pipe but slightly beyond the end. This is because air molecules have maximum freedom to vibrate a little outside the open end and not just at the open end.

The distance between the actual position of the antinode and the open end of the pipe is called the end correction.



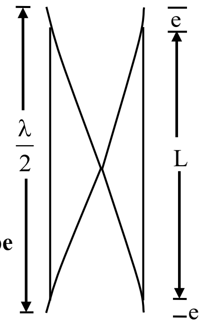
Closed pipe

In the case of a closed pipe, $\frac{\lambda}{4} = L + e$

and for an open pipe, $\frac{\lambda}{2} = L + 2e$

The end correction (e) depends on the diameter D of the closed pipe.

According to Rayleigh, $\mathbf{e = 0.3\ D}$

Open pipe


Expression for velocity of sound in terms of resonating lengths

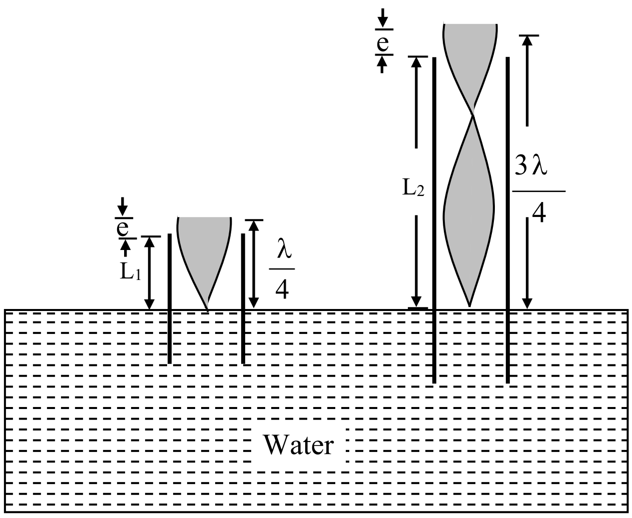
Consider an open pipe with its lower end under water. At a suitable minimum length L_1 of the pipe above the surface of water, the frequency of the air column becomes equal to the frequency of the excited tuning fork held at its mouth, leading to resonance. This occurs when

$$L_1 + e = \frac{\lambda}{4} \quad \ldots (1)$$

where e = end correction and λ = wavelength.

The second resonance occurs when the length of the air column is nearly thrice L_1.

$$L_2 + e = \frac{3\lambda}{4} \quad \ldots (2)$$



From equations [1] and [2]

$$L_2 - L_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} \therefore \lambda = 2(L_2 - L_1)$$

Since $v = f\lambda$, we have

$\boxed{v = 2f(L_2 - L_1)}$ v – speed of sound

where f = frequency of the air column = frequency of the tuning fork.

- Neglecting the end correction in equation [1] we get $\lambda = 4L_1$. Hence, $v = f\lambda = 4f L_1$. Thus, the approximate value of the velocity of sound in air at room temperature can be calculated.
- When the difference between two resonating lengths L_1 and L_2 are taken, the end correction e gets eliminated and hence the velocity of sound calculated is more accurate.
- Using (1) in (2) for $\lambda/4$, we get $L_2 + e = 3(L_1 + e)$ or $e = \frac{L_2 - 3L_1}{2}$.

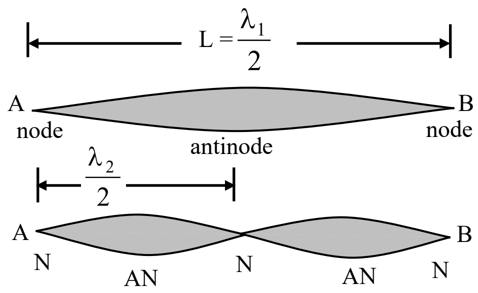
Since v varies with temperature, for a given frequency, L_1 also varies with temperature.

Stationary waves in a stretched string

Consider a uniform string of length Lof negligible cross section stretched between two fixed points Aand Bas shown. If Mis the mass of the string, then its linear density $m = \frac{M}{L}$

When the string is set into vibrations, transverse vibrations are set up in the string and they travel with a velocity v.

$v = \sqrt{\frac{T}{m}}$, where Tis the tension in the string.



These waves get reflected at the fixed end and form a stationary wave pattern. In the simplest mode of vibration, a single antinode is formed at the centre.

If L is the length and λ_1 is the wavelength, then

$L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$

If f_1 is the fundamental frequency, then $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

Since $v = \sqrt{\frac{T}{m}}$, we get $f_1 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

In the next mode of vibration, $\lambda_2 = L$ and hence, $f_2 = 2\left(\frac{1}{2L}\right)\sqrt{\frac{T}{m}}$

In general $f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$

The lowest frequency is called the **fundamental frequency.** The frequencies of the remaining modes called **harmonics** are the integral multiples of the fundamental frequency.

Laws of vibration of stretched strings

1. **Law of length:** The fundamental frequency of transverse vibrations of a stretched string is inversely proportional to its length (L) when the tension and linear density (m) remain constant.

   $\therefore\ \mathbf{f_1 \propto \frac{1}{L}}$, when T and m are constant.

2. **Law of tension:**The fundamental frequency of transverse vibration of a stretched string is directly proportional to the square root of the tension (T) when length and linear density (m) remain constant.

   $\therefore\ \mathbf{f_1 \propto \sqrt{T}}$, when L and m are constant.

3. **Law of mass:** The fundamental frequency of transverse vibration of a stretched string is inversely proportional to the square root of its mass per unit length (m), when tension (T) and length (L) remain constant.

$\therefore\ \mathbf{f_1 \propto \frac{1}{\sqrt{m}}}$, when L and T are constant.

Combining, we get $f_1 \propto \frac{1}{L}\sqrt{\frac{T}{m}}$

For a string of circular cross section, linear density $m = \pi r^2 \rho$; ρ = density, r = radius of cross section. The equation for fundamental frequency can then be written as $f = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 \rho}}$ or $f = \frac{1}{2L}\sqrt{\frac{4T}{\pi d^2 \rho}}$; d = diameter i.e., $f \propto \frac{1}{r}$ when ρ is constant, i.e., for wires of same material.

Beats

The periodic waxing and waning of sound due to the superposition of waves of slightly different frequencies is called beats.

When two notes of nearly equal frequencies get superposed, at any given point of space the displacements add up and subtract periodically. When the resultant displacement at a point is $(A_1 + A_2)$, the intensity is proportional to $(A_1 + A_2)^2$. This is the maximum intensity and this corresponds to the waxing. When the resultant displacement at a point is $(A_1 - A_2)$, the intensity is proportional to $(A_1 - A_2)^2$. This is the minimum intensity and this corresponds to the waning.

It can be shown that when two waves represented by equations $y_1 = A \sin 2\pi f_1 t$ and $y_2 = A \sin 2\pi f_2 t$ superpose, the equation for the resultant wave is given by

$$\mathbf{y} = \left\{2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right\}\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

It can thus be seen that the intensity varies between the maximum and the minimum at a frequency equal to the difference between the frequencies of the two waves.

If f_1 and f_2 are the frequencies of the two tuning forks sounded together, then the beat frequency (f_b) is given by $\boxed{f_b = f_1 \sim f_2}$

Beat frequency is equal to the difference in the frequencies of the two superposed waves.

If the beat frequency is more than 10 Hz, they are generally not perceptible

Uses of beats

The phenomenon of beats can be used to

- determine the unknown frequency of a tuning fork using a fork of known frequency
- tune musical instruments and
- tune radio receivers (super heterodyne receiver).

If the prongs of a tuning fork are filed, the frequency increases. If the prongs of a tuning fork are loaded, the frequency decreases.
If a number of tuning forks are arranged with common difference in frequency between two successive tuning forks, which produce beats of frequency f_b then the frequency of n^{th} fork $f_n = f_1 + (n - 1) f_b$
f_1 = frequency of the first tuning fork.

Illustrations

1. A cylindrical resonance tube open at both ends has a fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. Then the fundamental frequency of the air column becomes

(A) f (B) 2f (C) 3f (D) f/2

Ans (A)

When the tube is open at both ends, $f = \dfrac{v}{2l}$

When the tube is in water, $f_1 = \dfrac{v}{4\left(\dfrac{l}{2}\right)} = \dfrac{v}{2l} = f$

2. A resonating column of air contains

(A) stationary longitudinal waves
(B) stationary transverse waves
(C) transverse progressive waves
(D) longitudinal progressive waves

Ans (A)

3. A resonance air column of length 40 cm resonates with a tuning fork of frequency 450 Hz. Ignoring end correction, the velocity of sound in air will be (closed pipe)

(A) 720 ms^{-1}
(B) 820 ms^{-1}
(C) 920 ms^{-1}
(D) 1020 ms^{-1}

Ans (A)

$f = \dfrac{v}{4l}$ or $v = f4l = 450 \times 4 \times 40 \times 10^{-2}$ $\therefore$ $v = 720\ ms^{-1}$

4. The frequency of vibration of a string can be increased by

(A) increasing the length of the string keeping the tension constant.
(B) decreasing the density of the string keeping the tension constant.
(C) increasing the thickness of the string keeping the length constant.
(D) decreasing the tension of the string keeping the length constant.

Ans (D)

5. Transverse waves are generated in two uniform wires A and B by attaching their free ends to a vibrating source of frequency 600 Hz. The diameter of wire 'A' is one-third that of wire 'B' and tension in the wire 'A' is double that in wire B. What is the ratio of the velocities of waves in wire A and B?

(A) $\sqrt{3}:2$
(B) $2:\sqrt{3}$
(C) $3:\sqrt{2}$
(D) $\sqrt{2}:3$

Ans (D)

Velocity of a transverse wave in a wire

$v = \sqrt{\dfrac{T}{m}}$, m = mass per unit length

$v = \sqrt{\dfrac{T}{\pi r^2 \rho}} = \sqrt{\dfrac{4T}{\pi \rho d^2}}$

$v \propto \dfrac{\sqrt{T}}{d}$ $\therefore$ $\dfrac{V_A}{V_B} = \sqrt{\dfrac{T_A}{T_B}} \times \dfrac{d_B}{d_A} = \sqrt{\dfrac{2T_B}{T_B}} \times \dfrac{\dfrac{d_B}{3}}{d_B} = \dfrac{\sqrt{2}}{3}$

6. With the increase in temperature, the frequency of the sound from an organ pipe

(A) decreases
(B) increases
(C) remains unchanged
(D) changes erratically

Ans (B)

7. A stretched string of length l fixed at both ends can sustain stationary waves of wave length λ given by

(A) $\lambda = \dfrac{n^2}{2l}$
(B) $\lambda = \dfrac{l^2}{2n}$
(C) $\lambda = \dfrac{2l}{n}$
(D) $\lambda = 2ln$

Here n is a whole number.

Ans (C)

Let there be n loops in the string then, $n\frac{\lambda}{2} = l$ or $\lambda = \frac{2l}{n}$

8. An open and closed organ pipe have the same length. The ratio of the p^{th} mode of frequency of vibration of the two pipes is

(A) 1 (B) p (C) p(2pH) (D) $\frac{2p}{2p-1}$

Ans (D)

p^{th} mode of vibration of open organ pipe $n_1 = p\frac{v}{2l}$ and of closed organ pipe is,

$$n_2 = \left(\frac{2p-1}{2}\right)\frac{v}{2l} \Rightarrow \frac{n_1}{n_2} = \frac{2pv/2l}{(2p-1)v/2l} = \frac{2p}{2p-1}$$

9. A closed end organ pipe has a frequency f. If its length is doubled and radius is halved, its frequency will nearly become

(A) $\frac{f}{3}$ (B) $\frac{f}{2}$ (C) f (D) 2f

Ans (C)

$f \propto \frac{1}{l}$, no effect of radius.

10. Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of tuning forks (speed of sound = 340 m s^{-1}).

Solution

The frequency of tuning fork moving towards observer is

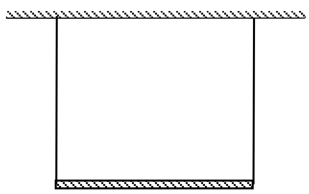
$f_1 = f\left(\frac{v}{v - v_s}\right)$ and the other $f_2 = f\left(\frac{v}{v + v_s}\right)$ given $f_1 - f_2 = 3 = fv\left(\frac{1}{v - v_s} - \frac{1}{v + v_s}\right)$

$$3 = \frac{2 f v v_s}{v^2\left(1 - \frac{v_s^2}{v^2}\right)} \Rightarrow \frac{2\left(\frac{v_s}{v}\right)f}{1 - \left(\frac{v_s}{v}\right)^2}.$$

If $v_s << v$ then $1 - \left(\frac{v_s}{v}\right)^2 \cong 1$ $\therefore 2\frac{v_s}{v}f = 3$

$v_s = \frac{3}{2} \times \frac{340}{340} = 1.5$ m s^{-1}.

11. A uniform horizontal rod of length 0.4 m and mass 1.2 kg is supported by two identical wires as shown. Where should a mass of 4.8 kg be placed on the rod, so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone (g = 10 m s^{-2})



Solution

$$\frac{1}{2l_1}\sqrt{\frac{T_1}{\mu}} = 2\left(\frac{1}{2l_1}\sqrt{\frac{T_2}{\mu}}\right)$$

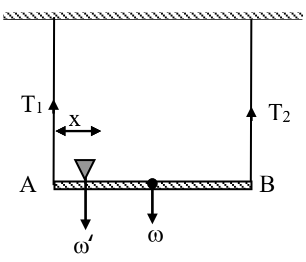
$\Rightarrow T_1 = 4T_2$ … (1)

Further $T_1 + T_2 = 4.8 + 1.2 = 6$ kg wt … (2)

From (1) and (2), $T_2 = 1.2$ kg wt

Taking moment about A, for equilibrium

Sum of clockwise moment = sum of anticlockwise moment



$\omega = 1.2$ kg wt

$\omega' = 4.8$ kg wt

$1.2 \times 0.2 + 4.8x = T_1 \times 0 + T_2 \times 0.4$

$\Rightarrow x = 0.5$ m = 5 cm

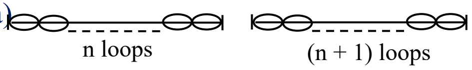
12. A string fixed at both the ends has consecutive standing wave modes for which the distances between adjacent nodes are 18 cm and 16 cm respectively.

(a) What is the minimum possible length of the string?

(b) If the tension in the string is 10 N and the linear mass density is 4 g m^{-1}, what is the fundamental frequency?

Solution

Let 'l' be the length of the string

(a)


$18n = l$ … (1), $16(n + 1) = l$ … (2)

From (1) and (2), $16(n + 1) = 18n$ i.e., $n = 8$ $\therefore l_{min} = 144$ cm

(b) $f_0 = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}} = 17.36$ Hz

13. A string 25 cm long and of mass 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats s^{-1} are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m s^{-1}, find the tension in the string.

Solution

Decreasing the tension in the string decreases beat frequency $\Rightarrow f_s > f_p$

$f_s - f_p = 8$

$$2\left(\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right) - \frac{v}{4l} = 8 \Rightarrow T = 27.04 \text{ N}$$

Doppler Effect

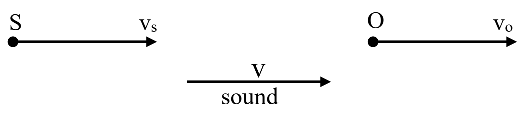
An observer on a railway platform listening to the whistle of an approaching train observes the pitch to be higher. When the train recedes, the pitch of the whistle appears to be lower. This effect is due to **Doppler effect.**

Doppler effect is the apparent change in the observed frequency of sound from a source due to the relative motion between the source and the observer.

Doppler effect is observed both in mechanical and electromagnetic waves.

In the case of mechanical waves such as sound waves, apart from the motion of source and the observer, motion of the medium (wind) also causes change in the frequency.

General expression for the apparent frequency



The apparent frequency f' heard by an observer due to the relative motion between a source and an observer as shown in the figure, is given by $\boxed{f' = \dfrac{f(v + w - v_o)}{(v + w - v_s)}}$ ...[1]

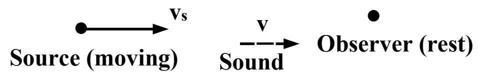
where **v** is the velocity of sound, **w** is the velocity of the wind or medium, $\mathbf{v_0}$ is the velocity of the observer, $\mathbf{v_s}$ is the velocity of the source and **f** is the frequency of the source.

The general formula for the Doppler frequency can be modified to specific cases as follows.

Case 1.Source in motion. The observer and the medium are at rest. i.e., $v_o = 0$, $w = 0$.

(a) Source moving towards the observer.

This means v_s is in the direction of v.Thus, v_s is positive.

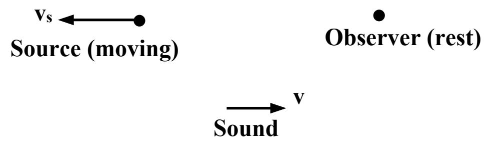


From equation [1] we get $\boxed{f' = f\left[\dfrac{v}{v - v_s}\right]}$ ...[2]

(b) Source moving away from the observer.

This means v_s is opposite to v. Thus, $v_s$is negative.

From equation[1] we get $\boxed{f' = f\left[\dfrac{v}{v + v_s}\right]}$ ...[3]



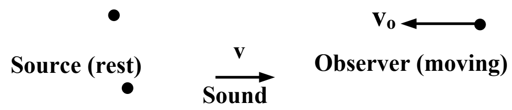
Case 2.Observer in motion. The source and the medium are at rest, i.e., $v_s = 0$, $w = 0$.

(a) Observer moving towards the source.

This means v_o is opposite to the direction of v.

Thus, v_o negative.

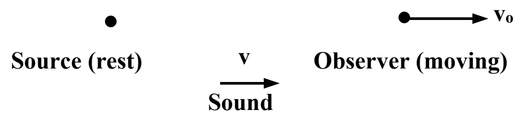
equation [1] we get $\boxed{f' = f\left[\dfrac{v + v_0}{v}\right]}$ ...[4]



(b) Observer moving away from the source.

This means v_o is in the direction of v. Thus, v_o is positive.

$\boxed{f' = f\left[\dfrac{v - v_0}{v}\right]}$ ...[5]



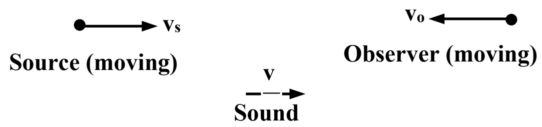
Case 3. Both source and observer in motion.

(a) Source and observer approaching each other;

$w = 0$, Source moves in the direction of v.

Thus, v_s is positive. Observer moves opposite to the direction of v. Thus, v_o is negative.

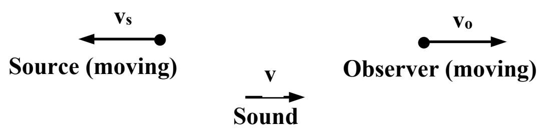
From equation [1] we get $\boxed{f' = f\left[\dfrac{v + v_0}{v - v_s}\right]}$ ...[6]



(b) Source and observer moving away from each other; $w = 0$,

Source moves opposite to v. Thus, v_s is negative.

Observer moves in the direction of v. Thus, v_o is positive.

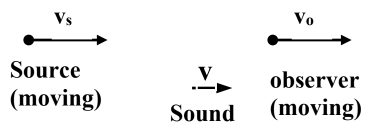


From equation [1] we get $\boxed{f' = f\left[\dfrac{v - v_0}{v + v_s}\right]}$ ...[7]

(c) Source and observer moving in the direction of v; w = 0

Both v_s and v_o are in the direction of v and are +ve.

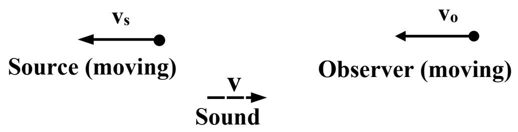
From equation (1), we get $\boxed{f' = f\left[\dfrac{v - v_0}{v - v_s}\right]}$ ...[8]



(d) Source and observer moving in the direction opposite to v; $\omega = 0$

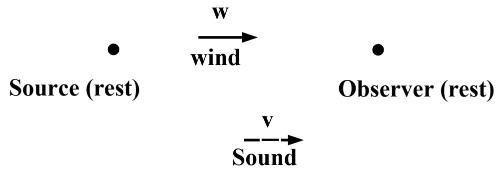
Both v_s and v_o are opposite to v and are –ve

From equation (1), we get $\boxed{f' = f\left[\dfrac{v+v_0}{v+v_s}\right]}$ …[9]



Case 4. Only medium in motion.

Since the observer and the source are at rest. $v_o = 0$, $v_s = 0$



From equation [1] we get $\boxed{f' = f\left[\dfrac{v+w}{v+w}\right]}$

Thus, $f' = f$ ...[10]

Applications of Doppler Effect

1. It is used in determining the velocities of aeroplanes and submarines.
2. It is used to determine the speeds of automobiles.
3. Doppler effect in light is used in determining the velocities of celestial bodies such as stars.
4. Bats perceive the change in frequency due to Doppler effect and hence use this in judging their velocities and also the velocities of objects around.

Illustrations

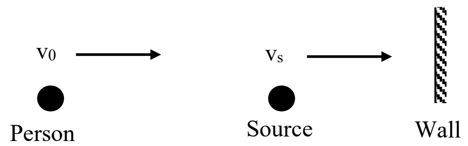
14. A source of sound of frequency f is moving towards a wall at a speed v_s. A person is following the source with a speed v_0. If v is the velocity of sound, obtain an expression for the beat frequency heard by the person.

Solution

Actual frequency of source = f

The direct frequency heard by the person, $f_1 = f\left(\dfrac{v+v_0}{v+v_s}\right)$ … (1)

The frequency heard at the wall $f' = f\dfrac{v}{v-v_s}$



∴ The reflected sound reaches the person.

The frequency of reflected sound

$$f_2 = f'\left(\frac{v+v_0}{v}\right) = \frac{fv}{v-v_s}\left(\frac{v+v_0}{v}\right) = f\left(\frac{v+v_0}{v-v_s}\right) \quad \text{… (2)}$$

∴ Beat frequency, $f_b = f_2 - f_1 = \left(\dfrac{v+v_0}{v-v_s} - \dfrac{v+v_0}{v+v_s}\right)f = \left[\dfrac{2v_s(v+v_0)f}{v^2 - v_s^2}\right]$

15. Doppler effect is independent of

(A) distance between source and listener
(B) velocity of source
(C) velocity of listener
(D) none of these

Ans (A)

Distance between source and listener

16. The fractional change in wavelength of light coming from a star is 0.014 %. What is its velocity?

(A) $4.2 \times 10^3 ms^{-1}$
(B) $3.8 \times 10^8 ms^{-1}$
(C) $3.5 \times 10^3 ms^{-1}$
(D) $4.2 \times 10^4 ms^{-1}$

Ans (D)

$\Delta\lambda = \dfrac{v}{c}\lambda$

$\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$

$\therefore\ v = \dfrac{0.014 \times 3 \times 108}{100}$

$v = 4.2 \times 10^4 ms^{-1}$

$$\frac{0.014}{100} = \frac{v}{c}$$

$$\therefore\ v = \frac{0.014}{100} \times c$$

17. A source and an observer are moving towards each other with a velocity equal to $\frac{v}{2}$, where v is the speed of sound The source is emitting sound of frequency f. The frequency heard by the observer will be

(A) 0 (B) $\frac{f}{3}$ (C) f^2 (D) 3f

Ans (D)

$$f' = \left(\frac{v+b}{v-a}\right)f \qquad \text{Given } a = b = \frac{v}{2}\ \therefore\ f' = \left(\frac{v+\frac{v}{2}}{v-\frac{v}{2}}\right)f = 3f$$

18. A racing car moving towards a cliff sounds the horn. The driver observes that the sound reflected form the cliff has a pitch one octave higher than the actual sound of the horn. If v is the velocity of sound, then velocity of car is

(A) $\frac{v}{3}$ (B) $\frac{v}{4}$ (C) $\frac{v}{\sqrt{2}}$ (D) $\frac{v}{2}$

Ans (A)

$$f' = 2f = \left(\frac{v+v_s}{v-v_0}\right)f \qquad v_s = v_0$$

$$\therefore\ 2 = \frac{v+v_0}{v-v_0} \qquad \therefore\ 2v - 2v_0 = v \;\; v_0$$

$$\therefore\ 3v_0 = v \qquad \therefore\ v_0 = \frac{v}{3}$$

19. A whistle revolves in a circle with an angular speed of 20 rad s^{-1} using a string of length 50 cm. If the frequency of sound form the whistle is 385 Hz, then what is the minimum frequency heard by an observer, which is far away from the centre in the same plane? ($v = 340\ ms^{-1}$)

(A) 333 Hz (B) 374 Hz (C) 385 Hz (D) 394 Hz

Ans (B)

$$f' = f\left(\frac{v-v_0}{v+v_s}\right)$$

$$f' = 385\left(\frac{340-0}{340+10}\right)$$

$$f' = 385 \times \frac{340}{350} = 374\ \text{Hz}$$

$v = rw = 10\ ms^{-1}$

20. The wavelength of the light coming from a distant star shifts towards the violet end of the spectrum. The star is

(A) at rest (B) moving towards

(C) moving away (D) may be moving towards or away from the earth

Ans (B)

21. A bus is moving with a velocity of 5 ms^{-1} towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air is 335 ms^{-1}, the number of beats heard per second by the passengers in the bus will be

(A) 3 (B) 4 (C) 5 (D) 6

Ans (C)

Due to Doppler's effect apparent frequency heard by observer when he is moving towards the source is

$$f' = \left(\frac{v+v_0}{v-v_s}\right)f = \left(\frac{335+5}{335-5}\right)165 = 170\ \text{Hz}$$

$\therefore$ Number of beats heard per second = f ′ – f = 170 – 165 = 5

22. A sound source is moving towards a stationary observer with $\left(\frac{1}{10}\right)$ of the speed of sound. The ratio of apparent to real frequency is

(A) $\frac{10}{9}$ (B) $\frac{11}{10}$ (C) $\left(\frac{11}{10}\right)^2$ (D) $\left(\frac{9}{10}\right)^2$

Ans (A)

$$f' = \left(\frac{v}{v-v_s}\right)f \qquad \therefore \frac{f'}{f} = \left(\frac{v}{v-v_0}\right) = \frac{10}{9}$$

23. Two cars are moving on two perpendicular roads towards a crossing with uniform speeds of 72 km hr^{-1} and 36 km hr^{-1}. If first car blows horn of frequency 280 Hz then the frequency of horn heard by the driver of second car when line joining the cars make 45° angle with the roads, will be

(A) 321 Hz (B) 298 Hz (C) 289 Hz (D) 280 Hz

Ans (B)

$$v_1 = \frac{72\times 5}{18} = 20\ \text{ms}^{-1},\quad v_2 = \frac{36\times 5}{18} = 10\ \text{ms}^{-1}$$

The components of velocities of the two cars along the direction of propagation of sound

= 20 cos 45° = 14.14 ms^{-1}and 10 cos 45° = 7.07 ms^{-1}

$\therefore$ Apparent frequency = $f' = f\left(\frac{v+v_2\cos 45°}{v-v\cos 45°}\right)$

$$\therefore f' = 280\left(\frac{330+7.07}{330-14.14}\right) = 298\ \text{Hz}$$

24. Two trains A and B are approaching a person each with a speed of 72 km h^{-1}. The train A produces a sound of frequency 500 Hz. Calculate the apparent frequency of sound heard by (i) the person and (ii) the driver in train B.

Solution

(i) $f' = f\left(\frac{v}{v-v_A}\right) = 500\left(\frac{330}{330-20}\right) = 532\ \text{Hz}$

(ii) $f' = f\left(\frac{v+v_B}{v-v_A}\right) = 500\left(\frac{330+20}{330-20}\right) = 564\ \text{Hz}$

25. Two tuning forks A and B are vibrating at the same frequency 256 Hz. A listener is standing midway between the forks. If both tuning forks move to the right with a velocity of 5 m s^{-1}, find the number of beats heard per second by the listener (v_{sound} in air = 330 m s^{-1})

Solution

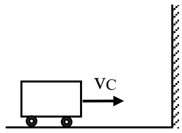
$$f_b = f'_a - f'_r = \left(\frac{v}{v-v_A}\right) - f\left(\frac{v}{v+v_B}\right) \approx 8\ \text{beats s}^{-1}$$

26. The driver of a car approaching a vertical wall notices that the frequency of the horn of car changes from 400 Hz to 450 Hz when it gets reflected from the wall. Find the speed of the car. (v_{Sound} = 340 m s^{-1})

Solution

Wall acts like a virtual source of frequency $f' = f\left(\dfrac{v}{v - v_C}\right)$

Driver of the car hears frequency $f'' = f'\left(\dfrac{v + v_C}{v}\right)$

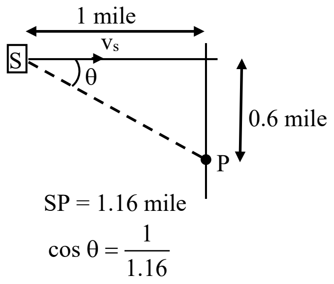


$$f' = f\left(\frac{v}{v - v_C}\right)\left(\frac{v + v_C}{v}\right)$$

$$450 = 400\left(\frac{340 + v_C}{340 - v_C}\right) \Rightarrow v_C = 20 \text{ m s}^{-1}$$

27. A locomotive approaching a crossing at a speed of 80 miles per hour sounds a whistle of frequency 400 Hz when if is at 1 mile from the crossing. There is no wind and the speed of sound in air is 0.2 mile/s. What frequency is heard by an observer 0.6 mile from the crossing on the straight road which crosses the rail road at right angles?

Solution



$v_s = 80$ mph

$v = 0.2$ miles $s^{-1} = 720$ mph

$$f' = f\left(\frac{v}{v - v_s\cos\theta}\right)$$

$$f' = 400\left(\frac{720}{720 - 80 \times \dfrac{1}{1.16}}\right) = 442 \text{ Hz}$$

SP = 1.16 mile

$\cos\theta = \dfrac{1}{1.16}$

28. A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m s^{-1}. How many beats per second will be heard if sound travels at a speed of 330 m s^{-1} by an observer

(i) between the wall and the source?

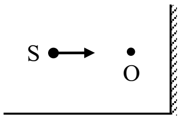
(ii) behind the source?

(iii) moving with the source?

Solution

(i) Frequency due to direct sound

$$f'_d = f\left(\frac{v}{v - v_s}\right) = 256\left(\frac{330}{330 - 5}\right) \approx 260 \text{ Hz}$$

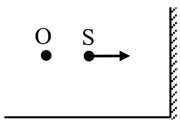


frequency due to reflected sound (wall acts like a virtual source of frequency $f\left(\dfrac{v}{v - v_s}\right)$

$$f'_r = f\left(\frac{v}{v - v_s}\right)$$

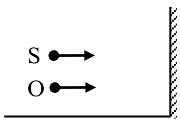
$\therefore$ beat frequency, $f_b = f'_d \sim f'_r = 0$

(ii) $f'_d = f\left(\dfrac{v}{v + v_s}\right) \approx 252 \text{ Hz}$



$f'_r = f\left(\dfrac{v}{v - v_s}\right) \approx 260 \text{ Hz} \therefore f_b = f'_d \sim f'_r \approx 8 \text{ beat s}^{-1}$

(iii) $f'_d = f = 256$ Hz



$$f_r' = f\left(\frac{v+v_0}{v-v_s}\right) \approx 264 \text{ Hz}$$

$\therefore\ f_b \approx 8$ beats s^{-1}

NCERT LINE BY LINE QUESTIONS

1. Some examples of wave motion are given in the following options. In which case wave motion is a combination of both transverse and longitudinal waves?

[NCERT XI Pg. 370)

(1) Motion of a kink in a longitudinal spring produced by displacing one end of the spring side ways
(2) Waves produced in a cylinder containing a liquid by moving its piston back and forth
(3) Waves produced by a motorboat sailing in water
(4) Both (1) and (3)

2. Longitudinal waves in a medium propagate due to [NCERT XIPg. 390]

(1) Shear modulus (2) Bulk modulus
(3) Both Shear and Bulk modulus (4) Young's modulus

3. Modification in Newton's formula for speed of sound in air was made by

[NCERT XI Pg. 376]

(1) Stefan (2) Boltzman (3) Laplace (4) Edison

4 . At what temperature will the speed of sound in air becomes 3 times of its value at 0°C?

[NCERT XIPg. 391]

(1) 1184°C (2) 1148*C (3) 2184°C (4) 2148°C

5. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, the wavelength of the reflected and transmitted sound are (speed of sound in air = 340 m/s and in water 1500 m/s) [NCERT XIPg. 391]

(1) 3.4 mm, 30 mm (2) 6.8 mm. 15 mm

(3) 0.34 mm, 1.5 mm (4) 6.8 mm. 30 mm

6. A pipe 30 cm long, is open at both the ends. Which harmonic mode of the pipe resonates with 1.1 kHz source? (v = 330 ms^1) [NCERT XIPg. 382]

(1) First (2) Second

(3) Third (4) Forth

7. A progressive wave is represented by y = 2 $\sin(100\pi t - 2\pi x)$, where x and y are in cm and ***t*** is in second. The maximum particle velocity and wave velocity respectively are [NCERT XIPg. 373]

(1) 628 cm/s, 628 cm/s (2) 50 cm/s. 50 cm/s

(3) 628 cm/s, 50 cm/s (4) 50 cm/s, 628 cm/s

8. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$. On reflection from a denser medium its amplitude becomes $\left(\frac{2}{3}\right)^{rd}$ of the amplitude of incident wave. The equation of reflected wave is [NCERT XIPg. 379]

(1) $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$ (2) $y = 0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

(3) $y = -0.4\sin 2\pi\left(t - \frac{x}{2}\right)$ (4) $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

9. A sound is produced by plucking a string in a musical instrument, then [NCERT XIPg. 381]

(1) The velocity of wave in string is equal to the sound velocity in string

(2) The frequency of wave in string is equal to the frequency of sound produced

(3) The wave in string is progressive

(4) The frequency of the wave in string is dcuble the frequency of sound

10. A glass tube of 100 cm length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500 Hz is brought at the upper end of the tube and the velocity of sound in air is 330 m/s, then the total number of resonances obtained will be (NCERT XI Pg. 382)

(1)4 (2)3 (3)2 (4) 1

11. A tuning fork *A* of frequency 512 Hz produces 5 beats per second when sounded with another tuning fork *B* of unknown frequency. If 0 is loaded with wax the number of beats is again 5 per second. The frequency of fork S before it was loaded is [NCERT XI Pg. 384)

(1) 507 Hz (2) 502 Hz (3) 517 Hz (4) 522 Hz

12. The equation of a stationary wave along a stretched string is given by $y = 5\sin\frac{2\pi x}{3}\cos 40\pi t$ in, where x and y are cm and t is in second. The separation between two adjacent nodes is [NCERT XI Pg. 379]

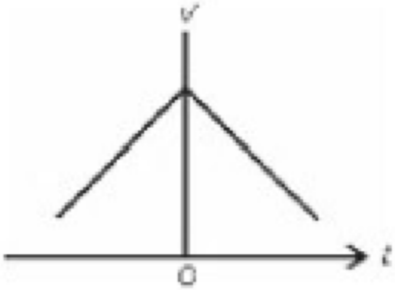
(1) 1.5 cm (2) 3 cm (3) 6 cm (4) 4 cm

13. A second harmonic has to be generated in a string of length ***L*** stretched between two rigid support. The point where the string has to be plucked and touched are [NCERT XI Pg. 381]

(1) Plucked at $\frac{L}{4}$ and touch at $\frac{L}{2}$ (2) Plucked at $\frac{L}{4}$ and touch at $\frac{L}{2}$
(3) Plucked at $\frac{L}{2}$ and touch at $\frac{L}{4}$ (4) Plucked at $\frac{L}{2}$ and touch at $\frac{3L}{4}$

14. An observer moves towards a stationary source of sound with a velocity one fifth of the velocity of sound. The percentage change in apparent frequency is
[NCERT XIPg. 386]
(1) 0% (2) 5% (3) 10% (4) 20%

15. A railway engine whistling at a constant frequency moves with a constant speed.lt goes past a stationary observer standing beside the railway track. The frequency (v') of the sound heard by observer is plotted against time *(t)*. Which of the following graph best represent the variation in apparent frequency with time?
[NCERT XI Pg. 385)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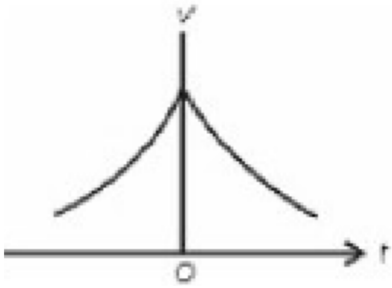
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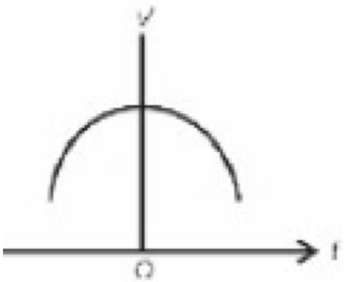
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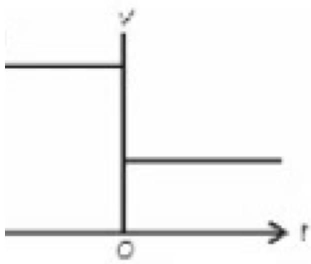

4)

16. If a wave is incident on a surface and a part of the incident wave is reflected back and a part is transmitted into the second medium, then [NCERT XI Pg. 378]
(1) Incident and refracted waves obey Snail's law of refraction
(2) Incident and refracted waves doesn't obey laws of refraction
(3) Incident and reflected waves obey the usual laws of reflection
(4) Both (1)and (3)

17. Two sitar strings A and B playing a note are slightly out of tune and produce beats of frequency 5 Hz. When the tension in the String B is slightly increased, the beat frequency is found to reduce to 3 Hz. If the frequency of String A is 427 Hz. The Original frequency of string B is [NCERT XIPg. 392]
(1) 422 Hz (2) 424 Hz (3) 430 Hz (4) 432 Hz

18. The transverse displacement of a string clamped at its both ends is given by $y = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$, where x and y are in metre and *t* is in second. The length of the string is 1.5 m and its mass is 3×10^{-2} kg. The tension in string is
[NCERT XI Pg. 392]

(1) 324 N (2) 648 N (3) 832 N (4) 972 N

19. In longitudinal stationary waves, displacement nodes are the points where there is [NCERT XI Pg. 379]

(1) Maximum displacement and maximum pressure
(2) Minimum displacement and minimum pressure change
(3) Minimum displacement and maximum pressure change
(4) Maximum displacement and maximum pressure change

20. Newton assumed that sound propagation in a gas takes under [NCERT XI Pg. 376]

(1) Isothermal condition (2) Adiabatic condition
(3) Isotropic condition (4) Isochoric condition

NCERT BASED PRACTICE QUESTIONS

1. Motion of air from one place to the other is known as -
(1) Sound wave (2) Transverse wave (3) Wind (4) SHM

2. Which of the following wave not required medium -
(1) Seismic wave (2) Water waves (3) Sound waves (4) X - rays

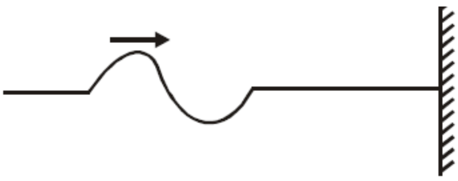
3. Which property of the medium helps to generate a mechanical wave in a medium or helps in energy transfer :-
(1) elastic forces (2) inertia (3) both (1) & (2) (4) none

4. Consider the following statements about sound passing through a gas.
(A)The pressure of the gas at a point oscillate in time
(B) The position of a small layer of the gas oscillate in time
(1) Both A & B are correct (2) A is correct but B is wrong
(3) B is correct but A is wrong (4) Both A & B are wrong

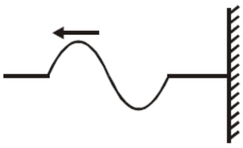
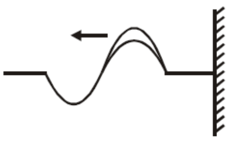
5. Which of the following wave can generate in spring.
(1) Only Longitudinal (2) Only transverse
(3) Both longitudinal & transverse (4) None

6. Which relation is correct for speed of sound wave in solid (s), liquid(l) & Gases(g) :-
(1) $V_s > V_l > V_g$ (2) $V_s < V_l < V_g$
(3) $V_s = V_l = V_g$ (4) $V_l > V_g > V_s$

7.



Which diagram represent the correct reflected wave.

(1)  (2) 

(3)  (4) 

8. Select the correct statements: -

(1) In standing wave amplitude is fixed at a given location but, it is different at different locations

(2) All particles vibrate in same phase in between successive nodes

(3) A stationary wave pattern resulting from super position of two travelling wave in opposite direction (same frequency waves)

(4) All the above

9. In which of the following wave Doppler shifts are same irrespective of whether the source moves or the observer moves towards each other.

(1) Ultrasonic (2) Infrasonic (3) Infrared (4) Audible

10. Doppler effect used in -

(1) sonography (2) echocardiogram

(3) astrophysics, military areas (4) all the above

11. When both wave superimpose completely then the energy is in the form of –

(1) Kinetic Energy

(2) Potential Energy

(3) Partial potential & partial K.E

(4) Energy will completely Loss

12. In a harmonic plane progressive wave which factor is different for all the particles in a wave.

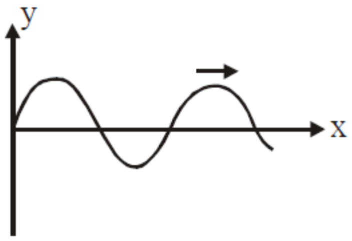
(1) Amplitude (2) Frequency (3) Phase (4) Time period

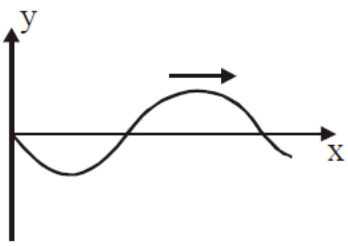
13. Relative to an observer at rest in a medium the speed of a mechanical wave in that medium doesn't depends on -

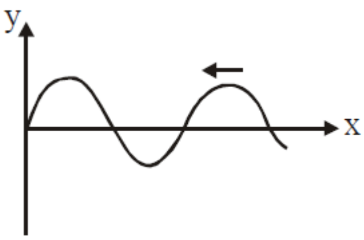
(1) Elasticity of medium (2) Mass of medium

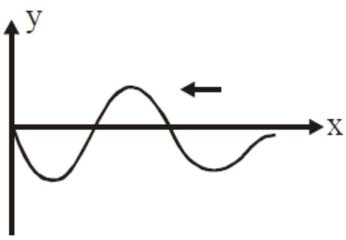
(3) Density of medium (4) Vel. of the source

14. Which wave represent the equation of wave $y = A \sin (kx - \omega t)$:-

(1) 

(2) 

(3) 

(4) 

15. A narrow sound pulse is sent across a medium.
The pulse have a definite -

(1) Frequency (2) Speed (3) Wave Length (4) Time Period

16. Which of the wave can't propagate in vacuum:-

(1) Radiowave (2) Seismic wave (3) X – rays (4) Light wave

17. Which of the wave is non-mechanical waves in nature.

(1) Wave on a string (2) Water waves

(3) X - Rays (4) Sound waves

18. Wave propagating on a streched string is:-

(1) Electromagnetic (2) Longitudinal

(3) Transverse (4) Non-Mechanical

19. Sound waves generated in a pipe filled with air by moving the piston up and down are :-

(1) Electromagnetic (2) Longitudinal (3) Transverse (4) Non-mechanical

20. In the medium steel which type of mechanical wave can propagate :-

(1) Only Transverse

(2) Only Longitudinal

(3) Both Transverse and Longitudinal

(4) None of the above

21. Which one is the displacement function for a longitudinal wave. (Where S is the displacement of a medium particle :-

(1) $S(x_1, t) = a \sin(\omega t - ky + \phi)$

(2) $S(x_1, t) = a \sin(\omega t - kx + \phi)$

(3) $S(y, t) = a \sin(\omega t - kx + \phi)$

(4) $S(x, t) = a \sin(\omega t - kz + \phi)$

22. A wave travelling along a string is described by $y(x, t) = 0.005 \sin\left(20\pi t - 10\pi x + \frac{\pi}{2}\right)$ cm. Where x is in cm and t is in sec. Calculate the displacement y of the wave at a distance x = 1 cm and time t = 1 sec.?

(1) y = 0.0025 cm (2) y = – 0.0025 cm (3) y = 0.005 cm (4) y = – 0.005 cm

23. Speed of sound in air at room temperature is approximately 343 m/s. What can be the speed of sound in Hydrogen at room temperature approximately.

(1) 1284 m/s (2) 2089 m/s (3) 686 m/s (4) 440 m/s

24. Find out correct statement :-

(1) A travelling wave or pulse suffers a phase change of $\frac{\pi}{2}$ on reflection at a rigid boundary

(2) A travelling wave or pulse suffers no phase change on reflection at an open boundary

(3) A travelling wave suffers same phase change from rigid and open boundary

(4) None of these

25. A pipe, 30 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? will resonance with the same source be observed if one end of the pipe is closed?
Take the speed of sound in air as 330 m/s.

(1) 4th Harmonic, yes (2) II Harmonic, yes

(3) 4th Harmonic, No (4) II Harmonic, No

26. Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produces beats of frequency 5Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3Hz. What is the original frequency of B if the frequency of A is 427 Hz?

(1) 422 Hz (2) 432 Hz (3) 424 Hz (4) 430 Hz

27. Find incorrect statement :-

(1) In a stationary wave, wavelength λ is twice the distance between two consecutive nodes

(2) In a progressive wave, wavelength λ is distance between two consecutive points of same phase at a given time.

(3) If k is angular wave number, wavelength λ is $\frac{\pi}{k}$.

(4) Frequency n of a wave is defined as 1/T and is related to angular frequency by $n = \dfrac{\omega}{2\pi}$

28. A pipe of length L with one end closed and other end open vibrates with frequencies given by :-

(1) $f = n\left(\dfrac{v}{2L}\right)$, n = 0, 1, 2, 3, ...

(2) $f = \left(n + \dfrac{1}{2}\right)\left(\dfrac{v}{2L}\right)$, n = 0, 1, 2, 3, 4 ...

(3) $f = (2n + 1)\dfrac{v}{2L}$, n = 1, 3, 5 ......

(4) $f = \dfrac{nv}{4L}$, n = 2, 4, 6 ......

29. Waves produced by a motorboat sailing in water :-
(1) Only transverse
(2) Only Longitudinal
(3) Transverse and Longitudinal
(4) None of these

30. Which of following waves are governed by Newton's laws of motion :-
(1) Mechanical waves
(2) Non-Mechanical waves
(3) Matter waves
(4) None of these

31. The picture of a progressive transverse wave at a particular instant of time gives -
(1) Shape of wave
(2) Motion of the particle of medium
(3) Velocity of wave
(4) None of above

32. The displacement of the wave given by equation $y(x, t) = a\sin(kx - \omega t + \phi)$ where $\phi = 0$ at point x and t = 0 is same as that at point:-

(1) $x + 2n\pi$ (2) $x + \dfrac{2n\pi}{k}$ (3) $kx + 2n\pi$ (4) None

33. Nature of ocean waves is :-
(1) Longitudinal (2) Transverse (3) Both (4) None

34. The waves on the surface of water are called as -
(1) P - wave (2) Shear wave (3) Both (4) None

35. If two sounds of very close frequencies 256 Hz and 260 Hz reach our ear simultaneously, then we hear a sound of frequency -
(1) 258 Hz (2) 4 Hz (3) 2Hz (4) None

36. Let a wave $y(x, t) = a\sin(kx - wt)$ is reflected from an open boundary and then the incident and reflected waves overlaps then amplitude of resultant wave -
(1) $2a\sin(kx)$ (2) $2a\cos(kx)$ (3) $2a\sin(kx/2)$ (4) $a\sin(kx)$

37. When a high - pressure pulse of air travelling down an open pipe reaches the other end then
(1) High pressure pulse starts travelling up the pipe
(2) Low pressure pulse starts travelling up the pipe
(3) Normal pressure pulse starts travelling up the pipe

(4) None of these

38. During Propagation of a plane progressive mechanical wave, which of the following statement is incorrect:-

(1) All the particles are vibrating in the same phase
(2) Amplitude of all the particles is equal
(3) Particles of the medium executes SHM
(4) Wave velocity wavelength depends upon the nature of the medium

39. Which of the following statements are false for stationary wave :-

(1) All the particles cross their mean position at the same time
(2) All the particles are oscillating with same amplitudes
(3) There is no energy transfer across any plane
(4) There are some particles which are always at rest

40. When the observer moves towards stationary source then which of the following is true regarding frequency and wavelength of wave observed by the observer -

(1) More frequency, less wavelength
(2) More frequency, more wavelength
(3) Less frequency, more wavelength
(4) More frequency, constant wavelength

41. In a sound wave, a displacement node is a pressure - antinode because -

(1) Pressure and density are maximum at displacement node
(2) Pressure and density are minimum at displacement node
(3) Change in pressure and change in density are maximum at displacement node
(4) None

42. A meter long tube (open at one end) with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 76.5 cm. Estimate the speed of sound in air (Ignore edge effect) -

(1) 347 m/s (2) 333 m/s (3) 330 m/s (4) 370 m/s

43. The transverse stationary wave on a stretched wire (Clamped at its both ends) is given by -y(x, t) = 0.06 $\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$ where x and y are in m and t in second. The length of string is 1.5 m and it's mass is 3×10^{-2}kg then all the points on the wire vibrate with -

(1) Same phase (2) Same amplitude
(3) Same energy (4) Different frequency

TOPIC WISE PRACTICE QUESTIONS

Topic 1: Basic of Mechanical Waves, Progressive and Stationary Waves

1. When a sound wave goes from one medium to another, the quantity that remains unchanged is

(a) frequency (b) amplitude (c) wavelength (d) speed

2. The equation of a spherical progressive wave is

(a) $y = a\sin\omega t$ (b) $y = a\sin(\omega t - kr)$ (c) $y = \frac{a}{\sqrt{2}}\sin(\omega t - kr)$ (d) $y = \frac{a}{r}\sin(\omega t - kr)$

3. The equation of plane progressive wave motion is $y = a\sin\frac{2\pi}{\lambda}(vt - x)$. Velocity of the particle is

(a) $y\frac{dv}{dx}$ (b) $y\frac{dy}{dx}$ (c) $-y\frac{dv}{dx}$ (d) $-v\frac{dy}{dx}$

4. The relation between velocity of sound in a gas (v) and r.m.s. velocity of molecules of gas ($v_{r.m.s.}$) is

(a) $v = v_{r.m.s.}(\gamma/3)^{1/2}$ (b) $v_{r.m.s.} = v(2/3)^{1/2}$ (c) $v = v_{r.m.s.}$ (d) $v = v_{r.m.s.}(3/\gamma)^{1/2}$

5. The equation Y = 0.02 sin (500 π t) cos (4.5 x) represents

(a) progressive wave of frequency 250 Hz along x-axis

(b) a stationary wave of wavelength 1.4 m

(c) a transverse progressive wave of amplitude 0.02 m

(d) progressive wave of speed of about 350 m s^{-1}

6. The equation $y = a\sin\frac{2\pi}{\lambda}(vt - x)$ is expression for

(a) stationary wave of single frequency along x-axis

(b) a simple harmonic motion

(c) a progressive wave of single frequency along x-axis

(d) the resultant of two SHMs of slightly different frequencies

7. The velocity of sound in hydrogen is 1224 m/s. Its velocity in a mixture of hydrogen and oxygen containing 4 parts by volume of hydrogen and 1 part oxygen is

(a) 1224 m/s (b) 612 m/s (c) 2448 m/s (d) 306 m/s

8. Sound waves of length λ travelling with velocity v in a medium enter into another medium in which their velocity is 4v. The wavelength in 2nd medium is

(a) 4λ (b) λ (c) $\lambda/4$ (d) 16λ

9. When a sound wave of frequency 300 Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. The maximum velocity of the particle is equal to

(a) 60π cms^{-1} (b) 30π cms^{-1} (c) 30 cms^{-1} (d) 60 cms^{-1}

10. A point source emits sound equally in all directions in a non–absorbing medium. Two points P and Q are at distances of 2 m and 3 m respectively from the source. The ratio of the intensities of the waves at P and Q is

(a) 3 : 2 (b) 2 : 3 (c) 9 : 4 (d) 4 : 9

11. A wave travelling in the +ve x-direction having displacement along y-direction as 1m, wavelength 2p m and frequency $\frac{1}{\pi}$ Hz is represented by

(a) $y = \sin(2\pi x - 2\pi t)$ (b) $y = \sin(10\pi x - 20\pi t)$ (c) $y = \sin(2\pi x + 2\pi t)$ (d) y = sin (x – 2t)

12. The equation of a stationary wave is $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$. The distance between a node and its next antinode is

(a) 7.5 units (b) 1.5 units (c) 22.5 units (d) 30 units

13. The velocity of sound in a container of air at –73ºC is 300 m/s. It temp. of container were raised to 127ºC, what would be the velocity of sound ?

(a) 300 m/s (b) $300\sqrt{2}$ m/s (c) $300/\sqrt{2}$ m/s (d) 600 m/s

14. At room temperature, velocity of sound in air at 10 atmospheric pressure and at 1 atmospheric pressure will be in the ratio

(a) 10 : 1 (b) 1 : 10 (c) 1 : 1 (d) cannot say

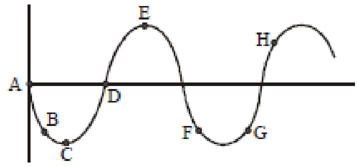
15. The speed of sound in air under ordinary conditions is around 330 m s^{-1}. The speed of sound in hydrogen under similar conditions will be (in m s^{-1}) nearest to

(a) 330 (b) 1200 (c) 600 (d) 900

16. A uniform wire of length 20 m and weighing 5 kg hangs vertically. If g = 10 m/s^2, then the speed of transverse waves in the middle of the wire is

(a) 10 m/s (b) 10 2 m/s (c) 4 m/s (d) zero

17. The diagram below shows the propagation of a wave. Which points are in same phase?



(a) *F* and *G* (b) *C* and *E* (c) *B* and *G* (d) *B* and *F*

18. A longitudinal wave is represented by $x = x_0 \sin 2\pi\left(nt - \frac{x}{\lambda}\right)$. The maximum particle velocity will be four times the wave velocity if

(a) $\lambda = \frac{\pi x_0}{4}$ (b) $\lambda = 2\pi x_0$ (c) $\lambda = \frac{\pi x_0}{2}$ (d) $\lambda = 4\pi x_0$

19. Two waves represented by $y_1 = a \sin \omega t$ and $y_2 = a \sin(\omega t + \phi)$ with $\phi = \frac{\pi}{2}$ are superposed at any point at a particular instant. The resultant amplitude is

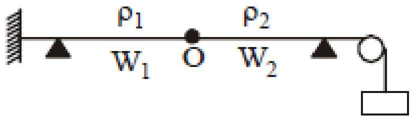
(a) a (b) 4a (c) $\sqrt{2}$ a (d) zero

20. A series of ocean waves, each 5.0 m from crest to crest, moving past the observer at a rate of 2 waves per second, what is the velocity of ocean waves?

(a) 2.5 m/s (b) 5.0 m/s (c) 8.0 m/s (d) 10.0 m/s

Topic 2: Vibration of String and Organ Pipe

21. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary waves is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is :



(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 4 : 1

22. The fundamental frequency of a closed end organ pipe is n. Its length is doubled and radius is halved. Its frequency will become nearly

(a) n/2 (b) n/3 (c) n (d) 2 n

23. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:

(a) 2f (b) f (c) f/2 (d) 3f/4

24. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

(a) 1.22v (b) 0.61v (c) 1.50v (d) 0.75v

25. In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x. The fraction of energy that is reflected is

(a) $\left[\frac{x-1}{x}\right]^2$ (b) $\left[\frac{x}{x+1}\right]^2$ (c) $\left[\frac{x-1}{x+1}\right]^2$ (d) $\left[\frac{1}{x}\right]^2$

26. A stretched wire 60 cm long is vibrating with its fundamental frequency of 256 Hz. If the length of the wire is decreased to 15 cm and the tension remains the same. Then the fundamental frequency of the vibration of the wire will be

(a) 1024 (b) 572 (c) 256 (d) 64

27. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is

(a) 105 Hz (b) 1.05 Hz (c) 1050 Hz (d) 10.5 Hz

28. The fundamental frequency of an open organ pipe is 300 Hz. The first overtone of this pipe has same frequency as first overtone of a closed organ pipe. If speed of sound is 330 m/s, then the length of closed organ pipe is
(a) 41 cm (b) 37 cm (c) 31 cm (d) 80 cm

29. Two wires are kept tight between the same pair of supports. The ratio of tension in the two wires are 2:1, radii are 3:1 and that of densities are 1:2. What is the ratio of their fundamental frequencies?
(a) 1:2 (b) 2:3 (c) 3:4 (d) 4:3

30. In the sonometer experiment, a tuning fork of frequency 256 Hz is in resonance with 0.4 m length of the wire when the iron load attached to free end of wire is 2 kg. If the load is immersed in water, the length of the wire in resonance would be (specific gravity of iron = 8)
(a) 0.37 m (b) 0.43 m (c) 0.31 m (d) 0.2 m

31. The fifth harmonic for vibrations of a stretched string is shown in figure. How many nodes are present here?

(a) 4 (b) 6 (c) 5 (d) 10

32. An organ pipe P_1, closed at one end vibrating in its first harmonic and another pipe P_2, open at both ends vibrating in its third harmonic, are in resonance with a given tuning fork. The ratio of the lengths of P_1 and $P_2$is :
(a) 8/3 (b) 1/6 (c) 1/2 (d) 1/3

33. A sonometer wire of length l vibrates in fundamental mode when excited by a tuning fork of frequency 416 Hz. If the length is doubled keeping other things same, the string will
(a) vibrate with a frequency of 416 Hz (b) vibrate with a frequency of 208 Hz
(c) vibrate with a frequency of 832 Hz (d) stop vibrating

34. Two strings A and B, made of same material, are stretched by same tension. The radius of string A is double of radius of B. A transverse wave travels on A with speed v_A and on B with speed v_B. The ratio v_A / v_B is
(a) 1/2 (b) 2 (c) 1/4 (d) 4

35. Where should the two bridges be set in a 110cm long wire so that it is divided into three parts and the ratio of the frequencies are 3 : 2 : 1 ?
(a) 20cm from one end and 60cm from other end
(b) 30cm from one end and 70cm from other end
(c) 10cm from one end and 50cm from other end
(d) 50cm from one end and 40cm from other end

36. If there are six loops for 1 m length in transverse mode of Melde's experiment, the no. of loops in longitudinal mode under otherwise identical conditions would be
(a) 3 (b) 6 (c) 12 (d) 8

37. A cylindrical tube open at both ends has a fundamental frequency n in air. The tube is dipped vertically in water so that half of it is immersed in water. The fundamental frequency of air column is
(a) n/2 (b) n (c) 2 n (d) 4 n

38. A fork of frequency 256 Hz resonates with a closed organ pipe of length 25.4 cm. If the length of pipe be increased by 2 mm, the number of beats/sec. will be
(a) 4 (b) 1 (c) 2 (d) 3

39. In a resonance column, first and second resonance are obtained at depths 22.7 cm and 70.2 cm. The third resonance will be obtained at a depth
(a) 117.7 cm (b) 92.9 cm (c) 115.5 cm (d) 113.5 cm

40. Vibrations are produced in a vertical tube of length 150cm closed at one end by a tuning fork of frequency 340Hz. Now water is filled slowly in the tube. If the speed of sound in air is 340 m/s then the minimum height of water required for resonance is
(a) 90cm (b) 75cm (c) 50cm (d) 25cm

41. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to

(a) 1 kg (b) 2 kg (c) 4 kg (d) 16 kg

Topic 3: Beats, Interference and Superposition of Waves

42. Beats are the result of
(a) diffraction
(b) destructive interference
(c) constructive and destructive interference
(d) superposition of two waves of nearly equal frequency

43. Two tones of frequencies n_1 and n_2 are sounded together. The beats can be heard distinctly when
(a) $10 < (n_1 - n_2) < 20$ (b) $5 < (n_1 - n_2) > 20$ (c) $5 < (n_1 - n_2) < 20$ (d) $0 < (n_1 - n_2) < 10$

44. Two travelling waves $y_1 = A \sin [k (x - ct)]$ and $y_2 = A \sin [k (x + ct)]$ are superimposed on string. The distance betweenadjacent nodes is
(a) $ct/$ p (b) $ct/$ 2p (c) p $/2k$ (d) p $/k$

45. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to the minimum intensity, is equal to
(a) 2 : 1 (b) 4 : 1 (c) 9 : 1 (d) 10 : 8

46. 10 forks are arranged in increasing order of frequency in such a way that any two nearest tuning forks produce 4 beats/sec. The highest frequency is twice of the lowest. Possible highest and the lowest frequencies (in Hz) are
(a) 80 and 40 (b) 100 and 50 (c) 44 and 22 (d) 72 and 36

47. 41 forks are so arranged that each produces 5 beats per sec when sounded with its near fork. If the frequency of last fork is double the frequency of first fork, then the frequencies (in Hz) of the first and the last fork are respectively.
(a) 200, 400 (b) 205, 410 (c) 195, 390 (d) 100, 200

48. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[2003]**
(a) (256 + 2) Hz (b) (256 – 2) Hz (c) (256 – 5) Hz (d) (256 + 5) Hz

49. Following are expressions for four plane simple harmonic waves.
(i) $y_1 = A\cos 2\pi\left(n_1 t + \frac{x}{\lambda_1}\right)$ (ii) $y_2 = A\cos 2\pi\left(n_1 t + \frac{x}{\lambda_1} + \pi\right)$
(iii) $y_3 = A\cos 2\pi\left(n_2 t + \frac{x}{\lambda_2}\right)$ (iv) $y_4 = A\cos 2\pi\left(n_2 t - \frac{x}{\lambda_2}\right)$
The pairs of waves which will produce destructive interference and stationary waves respectively in a medium, are
(a) (iii, iv), (i, ii) (b) (i, iii), (ii, iv) (c) (i, iv), (ii, iii) (d) (i, ii), (iii, iv)

50. Three sound waves of equal amplitudes have frequencies (n –1), n, (n + 1). They superpose to give beats. The number of beats produced per second will be :
(a) 3 (b) 2 (c) 1 (d) 4

51. The fundamental frequency of a sonometer wire of length ℓ is f_0. A bridge is now introduced at a distance of $\Delta\ell$ from the centre of the wire ($\Delta\ell << \ell$). The number of beats heard if both sides of the bridges are set into vibration in their fundamental modes is
(a) $\frac{8f_0\Delta\ell}{\ell}$ (b) $\frac{f_0\Delta\ell}{\ell}$ (c) $\frac{2f_0\Delta\ell}{\ell}$ (d) $\frac{4f_0\Delta\ell}{\ell}$

Topic 4: Musical Sound and Doppler's Effect

52. The loudness and pitch of a sound depends on
(a) intensity and velocity
(b) frequency and velocity
(c) intensity and frequency
(d) frequency and number of harmonics

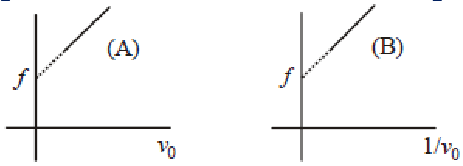
53. A whistle of frequency 385 Hz rotates in a horizontal circle of radius 50 cm at an angular speed of 20 radians s^{-1}. The lowest frequency heard by a listener a long distance away at rest with respect to the centre of the circle, given velocity of sound equal to 340 ms^{-1}, is

(a) 396 Hz (b) 363 Hz (c) 374 Hz (d) 385 Hz

54. A band playing music at frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, the expression for the beat frequency heard by the motorist is

(a) $\frac{v+v_m}{v+v_b}f$ (b) $\frac{v+v_m}{v-v_b}f$ (c) $\frac{2v_b\left(v+v_m\right)}{v^2-v_b^2}f$ (d) $\frac{2v_m\left(v+v_b\right)}{v^2-v_m^2}f$

55. A source of sound emits sound waves at frequency f_0. It is moving towards an observer with fixed speed v_s ($v_s< v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0, one of the following two graphs (A and B) will given the correct variation of the frequency f heard by the observer as v_0 is changed. The variation of f with v_0 is given correctly by :



(a) graph A with slope $=\frac{f_0}{\left(v+v_s\right)}$ (b) graph B with slope $=\frac{f_0}{\left(v-v_s\right)}$

(c) graph A with slope $=\frac{f_0}{\left(v-v_s\right)}$ (d) graph B with slope $=\frac{f_0}{\left(v+v_s\right)}$

56. A sound source emits frequency of 180 Hz when moving towards a rigid wall with speed 5 m/s and an observer is moving away from wall with speed 5 m/s. Both source and observer moves on a straight line which is perpendicular to the wall. The number of beats per second heard by the observer will be [Speed of sound = 355 m/s]

(a) 5 beats/s (b) 10 beats/s (c) 6 beats/s (d) 8 beats/s

57. Two trains move towards each other with the same speed. The speed of sound is 340 m/s. If the height of the tone of the whistle of one of them heard on the other changes 9/8 times, then the speed of each train should be

(a) 20 m/s (b) 2 m/s (c) 200 m/s (d) 2000 m/s

58. A train approaching a hill at a speed of 60 km/hour sounds a whistle of frequency 600 Hz when it is at a distance of 1 km from the hill. Wind is blowing in the direction of the train with a speed of 60 km/h. Find the frequency of the whistle heard by an observer on the hill: (Velocity of sound in air = 1200 km/h)

(a) 610 Hz (b) 620 Hz (c) 630 Hz (d) 650 Hz

59. Two trains are moving towards each other with speeds of 20m/s and 15 m/s relative to the ground. The first train sounds a whistle of frequency 600 Hz. The frequency of the whistle heard by a passenger in the second train before the train meets, is (the speed of sound in air is 340 m/s)

(a) 600 Hz (b) 585 Hz (c) 645 Hz (d) 666 Hz

60. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

(a) 0.5% (b) zero (c) 20 % (d) 5 %

NEET PREVIOUS YEARS QUESTIONS

1. A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of 27°C two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at 27°C is **[2018]**

(a) 330 m/s (b) 339 m/s (c) 300 m/s (d) 350 m/s

2. The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is **[2018]**

(a) 13.2 cm (b) 8 cm (c) 16 cm (d) 12.5 cm

3. Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a horn having a frequency 400 Hz. The frequency heard by the driver of the second car is [velocity of sound 340 m/s] :- **[2017]**

(a) 361 Hz (b) 411 Hz (c) 448 Hz (d) 350 Hz

4. The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? **[2017]**

(a) 20 Hz (b) 30 Hz (c) 40 Hz (d) 10 Hz

5. An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is : **[2016]**

(a) 66.7 cm (b) 100 cm (c) 150 cm (d) 200 cm

6. A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2 the ratio λ_2 / λ_1 is **[2016]**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_1+m_2}{m_2}}$ (c) $\sqrt{\frac{m_2}{m_1}}$ (d) $\sqrt{\frac{m_1+m_2}{m_1}}$

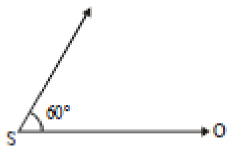
7. A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15 ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is: (Take velocity of sound in air = $330 ms^{-1}$) **[2016]**

(a) 765 Hz (b) 800 Hz (c) 838 Hz (b) 885 Hz

8. A string is stretched between two fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is : **[2015]**

(a) 205 Hz (b) 10.5 Hz (c) 105 Hz (d) 155 Hz

9. A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of $19.4 ms^{-1}$ at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer is (velocity of sound in air $330 ms^{-1}$) **[2015]**



(a) 103 Hz (b) 106 Hz (c) 97 Hz (d) 100 Hz

10. The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is **[2015]**

(a) 100 cm (b) 120 cm (c) 140 cm (d) 80 cm

11. A speeding motorcyclist sees traffic jam ahead of him. He slows down to 36 km/hour. He finds that traffic has eased and a car moving ahead of him at 18 km/hour is honking at a frequency of 1392 Hz. If the speeds of sound is 343 m/s, the frequency of the honk as heard by him will be : **[2014]**

(a) 1332 Hz (b) 1372 Hz (c) 1412 Hz (d) 1464 Hz

12. The number of possible natural oscillation of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are : (velocity of sound = $340 ms^{-1}$) **[2014]**

(a) 4 (b) 5 (c) 7 (d) 6

13. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by: **[2014]**

(a) $\frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}$ (b) $\frac{1}{\sqrt{n}}=\frac{1}{\sqrt{n_1}}+\frac{1}{\sqrt{n_2}}+\frac{1}{\sqrt{n_3}}$

(c) $\sqrt{n}=\sqrt{n_1}+\sqrt{n_2}+\sqrt{n_3}$ (d) $n=n_1+n_2+n_3$

14. The length of the string of a musical instrument is 90 cm and has a fundamental frequency of 120 Hz. Where should it be pressed to produce fundamental frequency of 180 Hz? **[NEET – 2020 (Covid-19)]**

(1) 75 cm (2) 60 cm (3) 45 cm (4) 80 cm

15. The distance covered by a particle undergoing SHM in one time period is (amplitude = A) :- **[NEET2019 (ODISSA)]**

(1) zero (2) A (3) 2A (4) 4A

16. A tuning fork with frequency 800 Hz produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at length 9.75 cm, 31.25 cm and 52.75 cm. The speed of sound in air is :- **[NEET – 2019 (ODISSA)]**

(1) 500 m/s (2) 156 m/s (3) 344 m/s (4) 172 m/s

17. In a guitar, two strings A and B made of same material are slightly out of tune and produce beats of frequency 6Hz. When tension in B is slightly decreased, the beat frequency increases to 7Hz. If the frequency of A is 530 Hz, the original frequency of B will be: **[NEET – 2020]**

1) 537 Hz 2) 523 Hz 3) 524 Hz 4) 536 Hz

18. If the initial tension on a stretched string is doubled, then the ratio of the initial and final speeds of a transverse wave along the string is [NEET–2022]

1) 1:1 2) $\sqrt{2}:1$ 3) $1:\sqrt{2}$ 4) 1: 2

NCERT LINE BY LINE QUESTIONS – ANSWERS

1)	d	2)	b	3)	c	4)	c	5)	c
6)	b	7)	c	8)	d	9)	b	10)	b
11)	c	12)	a	13)	a	14)	d	15)	d
16)	d	17)	a	18)	b	19)	c	20)	a

NCERT BASED PRACTICE QUESTIONS - ANSWERS

1) 3	2) 4	3) 3	4) 1	5) 3	6) 1	7) 1	8) 4	9) 3	10) 4
11) 1	12) 3	13) 4	14) 1	15) 2	16) 2	17) 3	18) 3	19) 2	20) 3
21) 2	22) 3	23) 1	24) 2	25) 4	26) 1	27) 3	28) 2	29) 3	30) 1
31) 1	32) 2	33) 3	34) 3	35) 1	36) 2	37) 2	38) 1	39) 2	40) 4
41) 3	42) 1	43) 1							

.

TOPIC WISE PRACTICE QUESTIONS - ANSWERS

1) 1	2) 4	3) 2	4) 1	5) 2	6) 3	7) 2	8) 1	9) 1	10) 3
11) 4	12) 1	13) 4	14) 3	15) 2	16) 1	17) 4	18) 3	19) 3	20) 4
21) 2	22) 1	23) 2	24) 1	25) 3	26) 1	27) 1	28) 1	29) 2	30) 1
31) 1	32) 2	33) 1	34) 1	35) 1	36) 1	37) 2	38) 3	39) 1	40) 4
41) 4	42) 3	43) 4	44) 4	45) 2	46) 4	47) 1	48) 3	49) 4	50) 2
51) 1	52) 3	53) 3	54) 3	55) 3	56) 1	57) 1	58) 3	59) 4	60) 3

NEET PREVIOUS YEARS QUESTIONS-ANSWERS

1) 2	2) 1	3) 3	4) 1	5) 3	6) 2	7) 3	8) 3	9) 1	10) 2
11) 3	12) 4	13) 1	14) 2	15) 4	16) 3	17) 3	18) 3		

NCERT BASED PRACTICE QUESTIONS - SOLUTIONS

1. A wave is not motion of matter as a whole ina medium.

2. X - rays are EMW so doesn't required medium.

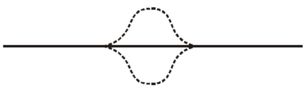
4. As the wave passes through gas it compress orexpands a small region of gas. This causechange in density of that region this changeinduce a change in pressure.

5. Spring is a solid material & in solid we cangenerate both type of waves.

6. Solid & Liquid are much more difficult tocompress than gases and so have much highervalue of Bulk Modulus. This factorcompensates for their higher densities thangases.

7. When wave reflects from denser medium orrigid boundary there is a phase change of π onreflection.

9. The answer 7 here Infrared waves areElectromagnetic waves.



11.

Net Displacement = 0

String becomes straight, No strain No. p.e

12. Phase is different because all particles receiveenergy at different times.

13.

$$v = \sqrt{\frac{\text{Elastic property}}{\text{inertial property}}}$$

$v = \sqrt{\frac{\text{Elastic property}}{\text{inertial property}}}$ not depends on vel. of source

15. The pulse does not have a definite wavelengthor frequency, but has a definite speed ofpropagation (in non - dispersive medium).

16. Seismic wave is a mechanical wave.

17. X-Rays are electromangetic waves.

18. String waves are mechanical Transverse waves.

19. Sound waves in air are Longitudinal.

22. y = 0.005 sin(20p – 10π + p/2)
= 0.005 in(10p + π/2)cm
= 0.005 sin(π/2)
y = 0.005 cm

23. $V = \sqrt{\frac{\gamma RT}{M_\omega}} \Rightarrow V\alpha \frac{1}{\sqrt{M_w}}$

25. For oop n_0=V/2l=550 Hz
So, 1.1 KHz will be II Harmonic and COP andOP of same length can't produce same frequency in any Harmonic case.

26. n_A ⊔ n_B = 5
by increasing the tension frequency willincrease
n_A ⊔ n_B ' = 3
If n_A = 427 Hz
So, that n_B = 422 Hz

30. Mechanical waves require inertia to propogate.

31. At a instant picture show position of medparticle of that instant. It shows shape of wave.

32. For displacement to be same

$$\sin(kx - \omega t) = \sin(kx^1 - \omega t)$$

$$\text{at } t = 0 \qquad kx + 2n\pi = kx'$$

$$x^1 = x + \frac{2n\pi}{k}$$

33. Longitudinal inside water and transverse onsurface of water.

34. P-wave inside water & S-wave on surface ofwater.

35. Frequency of resultant wave = $n_{avg} = \frac{n_1 + n_2}{2}$

36.

$$y_{in} = a\sin(kx - \omega t) = a\sin[-(\omega t - kx)]$$

$$y_{in} = -a\sin(\omega t - kx)$$

$$y_{ref} = -a\sin[\omega t + kx]$$

$$y_{sw} = y_i + y_r$$

$$= -a[\sin(\omega t - kx) + \sin(\omega t + kx)]$$

$$A_{sw} = 2a\cos kx$$

37. When compression reaches open end it reflectsas rarefraction.

38. In progressive wave Amplitude of all particlesof med will be same.

39. In standing wave amp. of diff. particle is diff.max. at Antinode and min at node.

40. Wavelength changes only when source moves.

41. Pressure wave shows change in pressure.
P - wave and disp. wave has N_2 phase diff.

42.

$\frac{\lambda}{4} = 25.5$ cm $\Rightarrow$ $\lambda = 102$ cm $= 1.02$ m

Now $U = n\lambda = (340)(1.02) \approx 347$ m/s

43. By, comparing λ = 3m, $l = \lambda$ /2 one loop isformed therefore, all particle will oscillate insame phase

TOPIC WISE PRACTICE QUESTIONS - SOLUTIONS

1. **(a)** Frequency of wave is a function of the source of waves.
Therefore, it remains unchanged.

2. **(d)** In the spherical source, the amplitude A of wave is inversely proportional to the distance r i.e., $A \propto \frac{1}{r}$ where r is distance of source from the point of consideration.

3. **(b)** $y = y_1 + y_2$

$$y = 2a\sin\frac{2\pi}{\lambda}vt.\cos\frac{2\pi}{\lambda}x \left[\because \sin(A-B)+\sin(A+B) = 2\sin A\cos B\right]$$

4. **(a)** Velocity of sound in a gas is

$v\sqrt{\gamma P/\rho}$ and from $P = \frac{1}{3}\rho v^2_{r.m.s.}$; $v_{r.m.s.} = \sqrt{3P/\rho}$ $\quad \therefore \frac{v}{v_{r.m.s.}} = \sqrt{\frac{\gamma}{3}}$

5. **(b)** Equation is of stationary wave. Comparing with the standard equation

$$y = 2A\sin\left(\frac{2\pi}{T}\right)t\cos\left(\frac{2\pi}{\lambda}\right)x \quad \frac{2\pi}{\lambda} = 4.5 \text{ or } \lambda = \frac{2\pi}{4.5} = 1.4\text{m}$$

6. **(c)** The equation of progressive wave propagating in the positive direction of X-axis is

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \text{ or } y = a\sin(\omega t - kx)$$

7. **(b)** If $\rho_H = 1$ then $\rho_{mix} = \frac{4\times1+1\times16}{(4+1)} = 4$

$$\frac{v_{mix}}{v_H} = \sqrt{\frac{\rho_H}{\rho_{mix}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}\text{ ;} \qquad v_{mix} = \frac{v_H}{2} = \frac{1224}{2} = 612\,\text{m/s}$$

8. **(a)** From $v = n\lambda$, we find $\lambda \propto v$ because freq. n is constant.
Therefore, new wavelength $= 4\lambda$.

9. **(a)** $a\omega = 0.1\times2\pi\times300 = 60\pi$ cm s^{-1}
For a wave travelling along positive x-axis

$$y = a\sin(\omega t - kx) = a\sin\left(2\pi nt - \frac{2\pi x}{\lambda}\right) = 0.2\sin2\pi\left(6t - \frac{x}{60}\right)$$

10. **(c)** Intensity = Energy/sec/unit area

$$\text{Area } r^2 \Rightarrow I \propto 1/r^2 \Rightarrow \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$$

11. **(d)** As $Y = A\sin(\omega t - kx + \phi)$

$$\omega = 2\pi f = \frac{2\pi}{\pi} = 2 \quad \left[\because f = \frac{1}{\pi}\right]$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi} = 1 \quad [\because \lambda = 2\pi]$$

$$\therefore Y = \sin(2t - x + \phi) \quad [\because A = 1\text{m}]$$

12. **(a)** $k = \frac{\pi}{15} \Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{15} \Rightarrow \lambda = 30$ units

Distance between node and next antinode $= \frac{\lambda}{4} = \frac{30}{4} = 7.5$ units

13. **(d)** $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

14. **(c)** At a given temperature, velocity of sound is not affected by pressure.

15. **(b)** $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

16. **(a)** Here, $m = \frac{5}{20}\,\text{kg/m} = \frac{1}{4}\,\text{kg/m}$

Tension in the middle of wire

T = weight of half the wire $= \frac{5}{2} \times g = \frac{5}{2} \times 10\,\text{N} = 25\,\text{N}$

As $v = \sqrt{T/m}$ $\therefore v = \sqrt{\frac{25}{1/4}} = 10\,\text{m/s}$

17. **(d)** The displacement of the points *B* and *F* are equal in magnitude and sign. So these points are in same phase.

18. **(c)** Particle velocity

$$v = \frac{d}{dt}\left[x_0 \sin 2\pi\left(nt - \frac{x}{\lambda}\right)\right] = 2\pi n x_0 \cos 2\pi\left(nt - \frac{x}{\lambda}\right)$$

$\therefore$ Maximum particle velocity = $= 2\pi n x_0$

Wave velocity $= \frac{\lambda}{T} = n\lambda$

Given, $2\pi n x_0 = 4n\lambda \Rightarrow \lambda = \frac{2\pi n x_0}{4n} = \frac{\pi x_0}{2}$

19. **(c)** Resultant amplitude is $\sqrt{a^2 + a^2}$ i.e., $\sqrt{2}a$

20. **(d)** Here $\lambda = 5.0$ m, $n = 2$

$\therefore v = n\lambda = 2 \times 5.0 = 10.0$ m/s

21. **(b)** $n_1 = n_2$

$T \rightarrow$ Same

$r \rightarrow$ Same

$l \rightarrow$ Same

Frequency of vibration $n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$

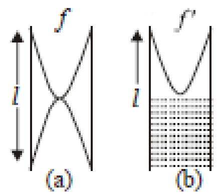
As T, r, and l are same for both the wires $n_1 = n_2$

$\frac{p_1}{\sqrt{\rho_1}} = \frac{p_2}{\sqrt{\rho_2}} \Rightarrow \frac{p_1}{p_2} = \frac{1}{2}$ $\because \rho_2 = 4\rho_1$

22. **(a)** Frequency does not depend upon radius. As length is doubled, fundamental frequency becomes half.

23. **(b)** The fundamental frequency in case (a) is $f = \frac{v}{2\ell}$

The fundamental frequency in case (b) is



$$f^{|} = \frac{v}{4(\ell/2)} = \frac{u}{2\ell} = f$$

24. **(a)** According to Hooke's law $F_R \propto x$

[Restoring force *FR* = *T*, tension of spring]

Velocity of sound by a stretched string $v = \sqrt{\frac{T}{m}}$

where *m* is the mass per unit length

Hence $v \propto \sqrt{T}$ or $\frac{v}{v^{|}} = \sqrt{\frac{T}{T^{|}}}$

or $v^{|} = v\sqrt{\frac{T^{|}}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22v$

25. **(c)** $\frac{A_1 + A_2}{A_1 - A_2} = x;\ \frac{A_2}{A_1} = \frac{x-1}{x+1}$; Energy $\propto A^2 \Rightarrow \left(\frac{x-1}{x+1}\right)^2$

26. **(a)** $L_0 = 60\,\text{cm} \quad \nu_0 = 256\text{Hz}$

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad \therefore \nu \propto \frac{1}{L}$$

$$\frac{\nu_1}{\nu_0} = \frac{L_0}{L_1} \Rightarrow \nu_1 = \nu_0\frac{L_0}{L_1} = 256 \times \frac{60}{15} = 1024\,\text{Hz}$$

27. **(a)** Given $\frac{nv}{2\ell} = 315$ and $(n+1)\frac{v}{2\ell} = 420$

The lowest resonant frequency is when n = 1

Therefore lowest resonant frequency = 105 Hz.

28. **(a)** For open pipe, $n = \frac{v}{2\ell}$, where $n_0$is the fundamental frequency of open pipe.

$$\therefore \ell = \frac{v}{2n} = \frac{330}{2 \times 300} = \frac{11}{20}$$

As freq. of 1st overtone of open pipe = freq. of 1stovertone of closed pipe

$$\therefore 2\frac{v}{2\ell} = 3\frac{v}{4\ell^{|}} \Rightarrow \ell^{|} = \frac{3\ell}{4} = \frac{3}{4} \times \frac{11}{20} = 41.25\,\text{cm}$$

29. (b)et ρ be the density of wire, and πr^2 be the area of the wire.

Thus, mass per unit length of wire $=\rho\times\pi r^2$

Thus, the ratio of mass per unit length of the wires is: $\frac{\rho_1}{\rho_2} \times \frac{r_1^2}{r_2^2} = \frac{1}{2} \times \frac{9}{1} = \frac{9}{2}$

Fundamental frequency of wire is given by $\frac{1}{21}\sqrt{\frac{T}{\mu}}$

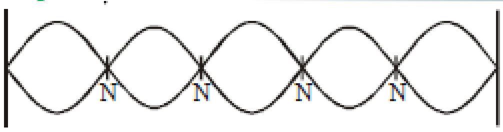
Since they are kept between same pair of supports, their lengths are equal.

Thus, $\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2} \times \frac{\mu_2}{\mu_1}} = \sqrt{\frac{2}{1} \times \frac{2}{9}} = 2:3$

30. **(a)** $\sqrt{T} \propto \ell \Rightarrow \sqrt{\frac{T_1}{T_2}} = \frac{\ell_1}{\ell_2} \Rightarrow \ell_2 = \ell_1 \times \sqrt{\frac{T_2}{T_1}}$

$$= \ell_1 = \sqrt{\frac{\rho_1 Vg - \rho_2 Vg}{\rho_1 Vg}} = \ell_1\sqrt{1 - \frac{\rho_2}{\rho_1}} = 0.4 \times \sqrt{1 - \frac{1}{8}} = 0.4 \times \sqrt{\frac{7}{8}} = 0.37$$

31. **(a)**



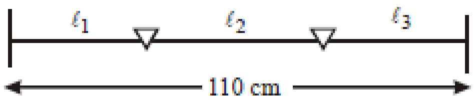
Total no. of nodes = 4

32. **(b)** $\frac{v}{4\ell_1}=\frac{3v}{2\ell_2}$ $\therefore \frac{\ell_1}{\ell_2}=\frac{1}{6}$

33. **(a)** Since Tension and mass per unit length remains unchanged, the frequency will be obtained in different mode.

34. **(a)** $\frac{v_A}{v_B}=\frac{D_B}{D_A}=\frac{1}{2}$

35. **(a)**



$n_1 : n_2 : n_3 = 3 : 2 : 1$

$n \propto \frac{1}{\ell}$

$\ell_1:\ell_2:\ell_3=\frac{1}{3}:\frac{1}{2}:\frac{1}{1}=2:3:6$

$\ell_1+\ell_2+\ell_3=110 \Rightarrow 2x+3x+6x=110 \Rightarrow x=10$

$\therefore$ The two bridges should be set at 2x i.e., 20 cm from one end and 6x i.e., 60 cm from the other end.

36. **(a)** No. of loops in longitudinal mode $=\frac{6}{2}=3$

37. **(b)** When tube is open, $n=\frac{v}{2\ell}$, where n is fundamental frequency of open organ pipe in air

When half of tube is dipped vertically in water, it behaves as a closed pipe of length $\frac{\ell}{2}$, so fundamental frequency n in this case is $\therefore n^{|}=\frac{v}{4(\ell/2)}=\frac{v}{2\ell}=n$

38. **(c)** $n_1=256=\frac{v}{4\ell_1}=\frac{v}{4\times 25.4}$ $\therefore v=256\times 101.6\,m/s$

$n_2=\frac{v}{4\ell_2}=\frac{256\times 101.6}{4\times 25.6}=254Hz$

No. of beats/sec = $n_1 - n_2 = 256 - 254 = 2$

39. **(a)** $\ell_1+x=\frac{\lambda}{4}=22.7$ equation (1)

$\ell_2+x=\frac{3\lambda}{4}=70.2$ equation (2)

$\ell_3+x=\frac{5\lambda}{4}$ equation (3)

From equation (1) and (2)

$x=\frac{\ell_2-3\ell_1}{2}\frac{70.2-68.1}{2}=\frac{2.1}{2}=1.05cm$

From equation (2) and (3) $\frac{\ell_3+x}{\ell_1+x}=5$

$\ell_3=5\ell_1+4x=5\times 22.7+4\times 1.05=117.7\,cm$

40. **(d)**

41. **(d)** Load supported by sonometer wire = 4 kg

Tension in sonometer wire = 4 g

If μ = mass per unit length

then frequency $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow 416 = \frac{1}{2l}\sqrt{\frac{4g}{\mu}}$

When length is doubled, i.e., $l' = 2l$

Let new load = L

As $\upsilon' = \upsilon$

$$\frac{1}{2\ell'}\sqrt{\frac{Lg}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{4g}{\mu}} \Rightarrow \frac{1}{4\ell}\sqrt{\frac{Lg}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{4g}{\mu}}$$

$\Rightarrow \sqrt{L} = 2 \times 2 \Rightarrow L = 16\,\text{kg}$

42. **(c)**Always keep in mind that the beat is defined as an interference pattern between two sounds of different frequencies. Also remember that the inference is different from beat as interference is produced when two waves of the same frequency travels in the same direction.

43. **(d)** As number of beats/sec = diff. in frequencies has to be less than 10, therefore $0 < (n_1 - n_2) < 10$

44. **(d)** $y = A\sin(kx - kct) + A\sin(kx + kct)$

$$= 2A\sin\left(\frac{kx - kct + kx + kct}{2}\right).\cos\left(\frac{kx - kct - kx - kct}{2}\right)$$

$= 2A\sin(kct).\cos kx$

Thus, $\frac{2\pi}{\lambda} = k, \quad \therefore \lambda = \frac{2\pi}{k}$

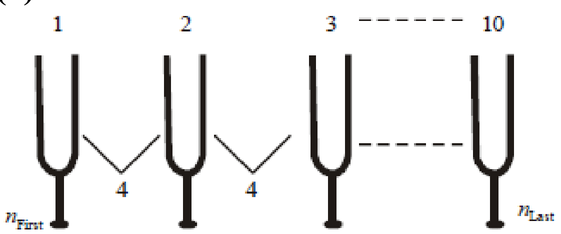
The distance between adjacent nodes $= \frac{\lambda}{2} = \frac{\pi}{k}$

45. **(b)** As intensity of wave $\propto$ (amplitude)^2

$\frac{I_1}{I_2} = \frac{9}{1} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{a_1}{a_2} = \frac{3}{1}$

$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{16}{4} \Rightarrow$ ratio is 4 : 1

46. **(d)**



Using $n_{Last} = n_{First} + (N - 1)x$

where N = Number of tuning forks in series

x = beat frequency between two successive forks

$\Rightarrow 2n = n + (10 - 1) \times 4 \Rightarrow n = 36$ Hz

47. **(a)** $n_{Last} = n_{First} + (N - 1)x$

$2n = n + (41 - 1) \times 5$

$\Rightarrow n_{First} = 200$ Hz and $n_{Last} = 400$ Hz

48. **(c)** A tuning fork of frequency 256 Hz makes 5 beats/ second with the vibrating string of a piano. Therefore,the frequency of the vibrating string of piano is (256 ± 5) Hz. i.e., either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. Now since the beat frequency decreases, we can conclude that the frequency of piano string is 251Hz

49. **(d)** In case of destructive interference

Phase difference f = 180° or π

So wave pair (i) and (ii) will produce destructive interference.

Stationary or standing waves will produce by equations (iii) & (iv) as two waves travelling along the same linebut in opposite direction.$n' = n + x$

50. **(b)** Maximum number of beats = $(\nu + 1) - (\nu - 1) = 2$

51. **(a)** $n_1 - n_2 = \dfrac{K}{\frac{1}{2}+\Delta l} - \dfrac{K}{\frac{1}{2}+\Delta l}$; $f_0 = \dfrac{K}{1}$

$$n_2 - n_2 = \frac{2K\Delta l}{\frac{l^2}{4}}(\Delta l << 1);\ = \frac{8f_0\Delta l}{1}$$

52. (c) Pitch is a term used to describe how high or low a note being played by a musical instrument or sung seems to be. It is dependent upon the frequency of source of sound.

Loudness depends upon the amplitude of sound wave. Thus it depends upon its intensity.

The larger the amplitude the more energy the sound wave contains therefore the louder the sound.

53. **(c)** Velocity of source

$$v_s = r\omega = 0.50 \times 20 = 10\,ms^{-1}$$

$$n^{|} = \frac{v}{v+v_s}n = \frac{340 \times 385}{340+10} = 374Hz$$

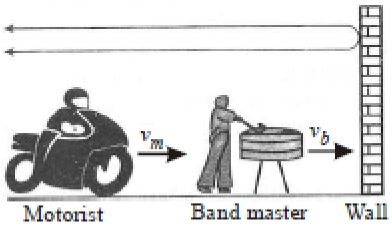
54. **(c)** The motorist receives two sound waves: direct one and that reflected from the wall.

$$f = \frac{v+v_m}{v+v_b}f$$

For reflected sound waves:

Frequency of sound wave reflected from the wall is

$$f^{||} = \frac{v}{v-v_b} \times f$$



Frequency of the reflected waves as received by the moving motorist is

$$f^{||} = \frac{v+v_m}{v} \times f^{||} = \frac{v+v_m}{v-v_b} \times f$$

Therefore, the beat frequency is

$$f^{|||} - f^{|} = \frac{v+v_m}{v-v_b} \times f - \frac{v+v_m}{v+v_b} \times f = \frac{2v_b(v+v_m)}{v^2 - v_b^2} f$$

55. (c) $f = f_0\left(\dfrac{v+v_0}{v-v_s}\right) = \dfrac{f_0}{v-v_s}v_0 + \dfrac{f_0 v}{v-v_s}$ where v is velocity of sound

As we see, this is of the form y=mx+c ;slope m is: $\dfrac{f_0}{v-v_s}$

56. **(a)** Frequency heard by observer directly coming fromsource $\dfrac{355-5}{355+5} \times 180 = 175Hz$

$f_2 \rightarrow$ frequency heard by observer after reflection

$$=\left[\frac{355}{355-5}\right]\times\left[\frac{355-5}{355}\right]\times 180=180\text{Hz}$$

$f_2 - f_1 = 5$ Hz

57. **(a)** Here, $v^{|}=\frac{9}{8}v$

Source and observer are moving in opposite direction, therefore, apparent frequency

$$v^{|}=v\times\frac{(v+u)}{(v-u)}\Rightarrow\frac{9}{8}v=v\times\frac{340+u}{340-u}$$

$$\Rightarrow u=\frac{340}{17}=20\,\text{m/sec}$$

58. (c) speed of sound in air is c and as the air is also moving the velocity of sound is c+v_a where v_a is the speed of air The train sounds the whistle when it is at distance x from the hill. Sound moving with velocity v with respect to ground, takes time t to reach the hill

$$t=\frac{x}{v}=\frac{x}{c+v_a}$$

after the reflection from the sound waves travels towards the train hence now the velocity of sound is $c-v_a$

let the distance travel by reflected wave be x′ therefore time taken by it $t^{|}=\frac{x}{c-v_a}$

the total time between echoing the whistle and its reflection t+t′ meanwhile the train travels the distance x–x′ with constant speed v_s

$$x-x'=(t+t')v_s$$

$$x-x'=\frac{v_s}{c+v_a}x+\frac{v_s}{c-v_a}x'$$

$$x\frac{c+v_a-v_s}{c+v_A}=x'\frac{c-v_a+v_s}{c-v_a}$$

$$x'=x\frac{(c-v_a)\times(c+v_a-v_s)}{(c+v_a)\times(c-v_a+v_s)}$$

$$x=\frac{(1200-60)\times(1200+60-60)}{(1200+60)\times(1200+60-60)}=630\text{Hz}$$

59. **(d)** $v^{|}=v\left(\frac{v+v_D}{v-v_S}\right)$

Here, $v=600$ Hz, $v_D=15$ m/s

$v_S=20$ m/s, $v=340$ m/s ; $\therefore v^{|}=600\left(\frac{355}{320}\right)\approx 666\text{Hz}$

60. **(c)** $n^{|}=n\left[\frac{v+v_0}{v}\right]=n\left[\frac{v+\frac{v}{5}}{v}\right]n\left[\frac{6}{5}\right]$; $\frac{n^{|}}{n}=\frac{6}{5};\frac{n^{|}-n}{n}=\frac{6-5}{5}\times 100=20\%$

NEET PREVIOUS YEARS QUESTIONS-EXPLANATIONS

1. **(b)** Two successive resonance are produced at 20 cm and73 cm of column length

$\therefore\frac{\lambda}{2}=(73-20)\times 10^{-2}\text{m}\Rightarrow\lambda=2\times(73-20)\times 10^{-2}$

Velocity of sound, $v = n\lambda$

$= 2 \times 320\,[73 - 20] \times 10^{-2} = 339.2\ \text{ms}^{-1}$

2. **(a)** For closed organ pipe, third harmonic

$$n = \frac{(2N-1)V}{4\ell} = \frac{3V}{4\ell}\ (\because N = 2)$$

For open organ pipe, fundamental frequency

$$n = \frac{NV}{2\ell} = \frac{V}{2\ell^{|}}\ (\because N = 1)$$

According to question, $\dfrac{3V}{4\ell} = \dfrac{V}{2\ell^{|}}$

$$\Rightarrow \ell^{|} = \frac{4\ell}{3\times 2} = \frac{2\ell}{3} = \frac{2\times 20}{3} = 13.33\,\text{cm}$$

3. **(c)** As we known from Doppler's Effect

$$f_{\text{apprent}} = f_0\left[\frac{v+v_0}{v-v_s}\right] = 400\left[\frac{340+16.5}{340-22}\right]$$

$$f_{\text{apprent}} = 448\text{Hz}$$

4. **(a)** Difference in two successive frequencies of closed pipe

$$\frac{2v}{4\ell} = 260 - 220 = 40\text{Hz or } \frac{2v}{4\ell} = 40\text{Hz} \Rightarrow \frac{v}{4\ell} = 20\text{Hz}$$

Which is the fundamental frequency of system of closedorgan pipe.

5. **(c)** For a closed organ pipe first minimum resonating length

$$L_1 = \frac{\lambda}{4} = 50\text{cm}$$

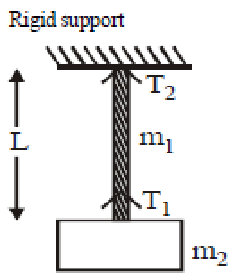
Next or second resonating length, $L_2 = \dfrac{3\lambda}{4} = 150\text{cm}$

6. **(b)** From figure, tension $T_1 = m_2 g$

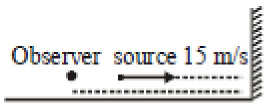
$T_2 = (m_1 + m_2)g$

As we know

$$\text{Velocity} \propto \sqrt{T} \text{ so, } \lambda \propto \sqrt{T} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1+m_2}{m_2}}$$



7. **(c)** According to Doppler's effect in sound



Apparent frequency, $n^{|} = \dfrac{v}{v - v_s} n_0$

$$= \frac{330}{330-15}(800) = \frac{330\times 800}{315} = 838\text{Hz}$$

The frequency of sound observer hears in the echoreflected from the cliff is 838 Hz.

8. **(c)** In a stretched string all multiples of frequencies can beobtained i.e., if fundamental frequency is *n*then higherfrequencies will be 2n, 3n, 4n ...

So, the difference between any twosuccessive frequencies will be 'n'

According to question, n = 420 – 315 = 105 Hz

So the lowest frequency of the string is105 Hz

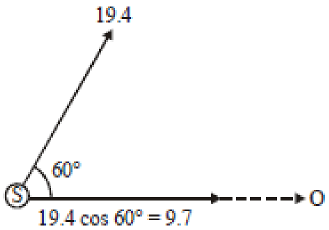
9. **(a)** Here, original frequency of sound, f_0 = 100 Hz

Speed of source V_S = 19.4 cos 60° = 9.7

From Doppler's formula

$$f^1 = f_0\left(\frac{V - V_0}{V - V_s}\right)$$

$$f^1 = 100\left(\frac{V - 0}{V - (+9.7)}\right)$$



$$f^1 = 100\frac{V}{V\left(1 - \frac{9.7}{V}\right)} = \frac{100}{\left(1 - \frac{9.7}{330}\right)} = 103\text{Hz}$$

Apparent frequency f^1 = 103 Hz

10. **(b)** Fundamental frequency of closed organ pipe $V_c = \frac{V}{4l_c}$

Fundamental frequency of open organ pipe $V_0 = \frac{V}{2l_0}$

Second overtone frequency of open organ pipe $= \frac{3V}{2l_0}$

From question, $\frac{V}{4l_c} = \frac{3V}{2l_0} \Rightarrow l_0 = 6l_c = 6 \times 20 = 120\text{cm}$

11. **(c)** According to Doppler's effect

Apparent frequency

$$n^{|} = n\left(\frac{v + v_0}{v + v_s}\right) = 1392\left(\frac{343 + 10}{343 + 5}\right) = 1412\text{Hz}$$

12. **(d)** In case of closed organ pipe frequency

$$f_n = (2n + 1)\frac{v}{4l}$$

for n = 0, f_0 = 100 Hz

n = 1, f_1 = 300 Hz

n = 2, f_2 = 500 Hz

n = 3, f_3 = 700 Hz

n = 4, f_4 = 900 Hz

n = 5, f_5 = 1100 Hz

n = 6, f_6 = 1300 Hz

Hence possible natural oscillation whose frequencies< 1250 Hz = 6(n = 0, 1, 2, 3, 4, 5)

13. **(a)** Total length of string $l = l_1 + l_2 + l_3$

(As string is divided into three segments)

But frequency $\propto \frac{1}{\text{length}}\left(\because f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}\right)$

So, $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

14. Frequency of stretched string

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$$

Here T and m are constant so $n \propto \frac{1}{\ell}$

Therefore $\frac{n^{|}}{n} = \frac{\ell}{\ell^{|}} \Rightarrow \frac{180}{20} = \frac{90}{\ell} \Rightarrow \ell^{|} = 60\text{cm}$

15. The distance moved is a scalar quantity and in a simple harmonic motion the distance covered in one time period is 4A. For example, if the particle is at the left extreme at t=0 then at t=T it will again come back to that point

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→ A O A

The distance covered =2A+2A =4A

16. For the tube open at one end, the resonance frequencies are $\frac{nv}{4l}$, where n is a positive odd integer. If the tuning fork has a f requency v and l_1,l_2,l_3 are the successive lengths of the tube in resonance with it, we have $\frac{nv}{4l_1} = v$

$$\frac{(n+2)v}{4l_2} = v,\ \frac{(n+4)v}{4l_2} = v$$

giving $l_3 - l_2 = l_2 - l_1 = $4v2v=2vv

By the question, $l_3 - l_2 = (52.75-31.25)\text{cm} = 21.50\text{cm}$

and $l_2 - l_1 = (31.25-9.57)\text{cm} = 21.50\text{cm}$

Thus, $\frac{v}{2v}$ =21.50cm

or $v = 2v \times 21.50\text{cm} = 2\times800\text{s}^{-1}\times21.50\text{cm} = 344\text{m/s}$.

17. $f_A - f_B = 6$

$530 - f_B = 6$

$f_B = 524Hz$

18. $V \propto \sqrt{T}$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \frac{1}{\sqrt{2}}$$